DIVISION OF THE HUMANITIES AND SOCIAL SCIENCES

CALIFORNIA INSTITUTE OF TECHNOLOGY

PASADENA, CALIFORNIA 91125

The Formation of Multiple Teams

Katerina Sherstyuk
California Institute of Technology

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Abstract

Organizational forms such as task-oriented teams have often been proposed as a method to enhance the efficiency of a firm. Under asymmetric information, however, the costs of acquiring the information needed to improve efficiency may outweigh the efficiency gains and lead to lower profits. We illustrate this idea by considering a profit-maximizing principal who needs to allocate a group of agents among a number of projects, given that the principal has incomplete information about the agents' abilities. We study feasible incentive-compatible (truth-revealing) individually rational mechanisms under both the dominant strategy and Bayesian Nash behavioral assumptions. Some attention is also paid to Nash equilibrium mechanisms. The paper covers derivation of optimal mechanisms, efficiency analysis, and analysis of the principal's expected profit as a function of different types of environment and information structures. We find that if the principal has little or no information about the agents' private characteristics and the agents follow dominant strategy behavior, the principal may often run into losses in an attempt to discover the hidden information. Paradoxically, the loss occurs when the efficiency gains from team production are high and the competition among the agents is low. If the hidden information about each agent can be summarized as a one-dimensional type parameter, and if a prior distribution function of the agents' types is common knowledge among the agents and the principal, an expected-profit maximizing Bayesian equilibrium mechanism exists and is of the optimal auction form (Myerson, 1981). Moreover, the mechanism can be equivalently implemented in dominant strategies with no expected profit loss for the principal. Yet, the principal's profit often decreases with an increase in the number of projects. These findings suggest that, in profit-maximizing firms with low competition among the employees, efficient organizational forms may often be foregone in favor of profits.

The Formation of Multiple Teams

Katerina Sherstyuk*

1 Introduction

The notion of a good organization as a robustly structured hierarchical unity has been recently changing in favor of the idea of more flexible organizational forms. With the growing diversity and complexity of tasks faced by a firm, one may expect high efficiency gains from the more flexible and responsive organizational structures. In particular, the notion of a "virtual corporation," which can be thought of as a flexible task-oriented partnership, has become quite popular in the literature (Byrne, Brandt, Port (1993)). One might ask whether flexible organizations should necessarily take the form of parterships, or whether owners of private firms can also gain from internal organizational flexibility. To answer this, one needs to consider the possible implications of flexible organizational forms, both for production efficiency and for owners' profit, as well as the informational requirements imposed on the management by the organization structure. In this study, we address the above question in the principal-agent framework and focus on one specific problem that a principal is likely to face in a flexible multi-project organization – the problem of forming temporary task-oriented teams from a given set of agents. Assuming the principal has incomplete information about the agents' characteristics, we consider whether she can gain from teams-based production. Curiously, we learn that rather often, when the potential efficiency gains from flexible organizations are high, it becomes too costly for the principal to run such organizations.

We consider an adverse selection model with a principal who has a number of (possibly profitable for her) projects to carry out and needs to hire the agents to work on these projects – each person for at most one project – from a given group of people. Each agent is characterized by his preference over working on projects (which might represent his personal taste and/or the difficulty of each job for him), and each subgroup of agents – by

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its productivity of working on each project. The principal needs to choose an allocation of the agents among projects that would maximize the principal's profit, which is the share of the revenue from all the projects net of the payments to the agents necessary to induce them to behave in the principal's interest. A difficulty arises when the principal does not have complete information about the agents' characteristics and hence cannot impose her most preferred outcome without the costly creation of the "right" incentives for the agents.

The recent literature in the economics of incomplete information pays a lot of attention to both the issue of inducing the "right" incentives for agents working in teams (Groves (1973), Holmstrom (1982), McAfee and McMillan (1991)), and the one of choosing the right agent for a job (Laffont and Tirole (1987), McAfee and McMillan (1987)). Both the moral hazard and the adverse selection aspects of the problem have been studied. Yet, the problem of optimally selecting teams for a given set of projects has been hardly addressed. Bolle's "Team Selection" paper (1991) is a rare study that addresses the issue of team formation in the Nash equilibrium framework with complete information. In our study, we use Bolle's approach to model the team production process, but consider incomplete information environments and seek to find optimal incentive-compatible team selection mechanisms under various behavioral assumptions. We use a simple model of team production where the moral hazard problem is absent. The social surplus produced by a team of agents on a project is a deterministic function of the agents' joint productivity parameter, which is assumed to be common knowledge, and the agents' private characteristics, or cost types. The agents' private characteristics are assumed to be independent.

We mainly focus on two information structures and two behavioral assumptions. In section 2 we assume a "complete ignorance" information structure, where there is no well-defined probability assessment over the agents' (possibly multi-dimensional) private characteristics that is common knowledge among the agents and the principal. Therefore, the agents are assumed to follow dominant strategy behavior. We consider feasible dominant strategy incentive compatible (DSIC) and individually rational (IR) mechanisms. Under these assumptions, the social efficiency maximizing information revelation mechanisms have been broadly studied in the literature, and truth is found to be a dominant strategy if and only if the mechanism is Groves-Clark (Groves (1973), Clark (1971), Green and Laffont (1977)). However, it is generically not budget-balancing (Green and Laffont (1977), Walker (1980)), and hence the "social planner" (the principal) may run into losses. The inconsistency of efficiency maximization with the principal's profit maximization is recognized by a number of authors (Groves and Loeb (1975, 1979), J. Miller and P. Murrell (1981), G. Miller (1992)), but the problem of profit-maximization constrained by dominant strategy incentive compatibility and individual rationality of the agents has not been explicitly studied. We address this problem and find that under the "complete ignorance" assumption, there does not exist a uniform strongly optimal mechanism for a principal – a mechanism that in any environment[1] produces a higher level of profit than any other DSIC IR mechanism. The mechanisms that are optimal under an alternative, weaker criterion of optimality (Arrow and Hurwicz, 1972) exist, but they can be "ad hoc," very inefficient and hardly sensitive to the environment. Among the efficiency maximizing mechanisms, any DSIC mechanism that is individually rational for the agents is not individually rational for the principal. The result indicates that, in contrast to the efficiency-maximization case, in generic environments a principal cannot successfully organize production without having substantial information about agents' characteristics.

We further analyze specific types of environments that can be of interest and find that the more competitive the environment is, the higher the principal's profit is. In this part, our results are very similar to Makowski and Ostroy (1987), who establish a close connection between perfect competition and efficient dominant strategy incentive compatible mechanisms. We find that in the perfectly competitive environments, where each agent is dispensable, the principal can use efficient DSIC mechanism to acquire the whole social surplus for herself. On the other hand, perhaps surprisingly, we establish that the higher the social gains from the teams' production are, the lower is the principal's profit. Thus, pursuing efficiency gains from a flexible organization form appears to be completely at odds with the principal's self-interest.

In section 3 we move to the Bayesian Nash equilibrium framework, where the information incompleteness on the principal's part is less extreme. The full consideration of this problem would require us to address a multi-dimensional adverse selection issue, allowing agents' costs to be independent (or stochastically correlated) across project. While a number of recent studies approach the latter problem (Rochet (1985), McAfee and McMillan (1988), Wilson (1993), Armstrong (1993a, 1993b)), all of them indicate that explicit characteristics of the solutions are difficult to obtain even for a single-agent case. In characterizing incentive-compatible mechanisms in a multi-dimensional setting, McAfee and McMillan (1988) derive a generalized single-crossing property which requires, essentially, the agents' types to line up. In this study, we reduce the problem to a one-dimensional case by assuming that agents' costs on different projects are deterministic functions of one-dimensional types. The types' probability distribution is known to the principal and the agents. Both the principal and agents are risk-neutral. Under these assumptions, we derive Bayesian incentive compatible (BIC) IR mechanisms that maximize the principal's expected profit, and present necessary and sufficient conditions for such mechanisms to exist. We show that an optimal BIC IR mechanism is similar to an optimal auction with risk-neutrality and independent private values a la Myerson (1981). Furthermore, following a technique presented by Mookherjee and Reichelstein (1992), we show that an optimal mechanism can be equivalently implemented in dominant strate-

[1]By an environment we mean the teams' productivity parameters and the agents' private cost characteristics.

gies with no expected profit loss to the principal. In this way, the problem of finding an optimal DSIC IR mechanism is resolved for this type of information structure. However, we find that the larger the number of projects, i.e., the more complicated the allocation problem is, the stronger the restrictions are on the mechanism needed to satisfy the agents' incentive compatibility. The principal is often bound to treat agents of different types alike and hence faces losses in her expected profits.

In section 4 we briefly explore the levels of profit achievable by the principal if she is "completely ignorant" (as in section 2), but the agents have complete information about each other's characteristics and follow Nash equilibrium behavior. We present our main conclusions and discuss their implications for issues about flexible organizational forms in section 5. Section 6 contains proofs of the propositions.

2 Dominant strategy mechanisms with "complete ignorance"

2.1 The model

Consider a simple case of the multiple team formation problem with pure adverse selection, where nature-induced uncertainty and moral hazard are absent. We are given the set of agents $N = \{1, ..n\}$, $n \geq 1$, and the set of projects $K = \{1, .., k\}$, $k \geq 1$, among which the agents are to be allocated. Each agent i is characterized by a vector of costs (disutility levels) $c_i = \{c_{ij}\}$, with each c_{ij} denoting i's disutility of being assigned to the project j. For every i, let $c_i \in \mathcal{C}_i$, where $\mathcal{C}_i$ is a convex bounded subset of R^k with a non-empty interior. We may interpret these disutilities as exogenously given costs of an agent's effort which vary depending on the project to which he is assigned. Let $C = (c_1, .., c_n)$ denote the matrix of all agents' costs, and C_{-i} – the matrix of costs of agents other than i, for every $i \in N$. Then let $\mathcal{C} = \times_i \mathcal{C}_i$ denote the set of all possible disutility profiles. Assume that the costs c_{ij} are expressed in monetary terms and are the private information of each respective agent.

A team of agents $T \subseteq N$ assigned to a project $j \in K$ is represented by an n-dimensional vector $x_j = (x_{1j}, .., x_{nj})$, where for all $i \in N$ $x_{ij} = 1$ if agent i is assigned to the project j, and $x_{ij} = 0$ otherwise. Then an $n \times k$ matrix $X = (x_1, .., x_k)$ denotes a particular allocation, or assignment, of agents across the projects. An allocation X is *feasible* if for all $i \in N$, $\sum_{j \in K} x_{ij} \leq 1$ and for all $i \in N$, $j \in K$, $x_{ij} \in \{0, 1\}$[2]. Let $\mathcal{X}$ denote the set of all feasible allocations.

[2] Alternatively, we could assume that an agent's contribution can be distributed among several projects and, therefore, x_{ij} is a continuous variable, $x_{ij} \in [0, 1]$. Most of the results presented in this paper are still valid in the continuous case. We will indicate which parts of the analysis hold for $x_{ij} \in \{0, 1\}$ case only.

Suppose each team x_j, $x_{ij} \in \{0,1\}$ for all $i \in N$ is characterized by its potential productivity on a project j, $F_j(x_j)$, $|F_j(x_j)| < \infty$, expressed in monetary terms. Assume $F_j(0,..,0) = 0$ for all j. For each x_j and each $j \in K$, let $\mathcal{F}_j(x_j) \subset R$ be the set of all possible productivity parameters. Assume that the projects have no external productivity effects on each other, and that each agent can be assigned to at most one project. Then for any feasible allocation X of agents among the projects, the total gross productivity of the allocation equals the sum of the teams' productivities over projects:

$$F(X) = \sum_{j \in K} F_j(x_j) \ .$$

Note that $F(0) = 0$. For future convenience, for every agent $i \in N$, let x_i denote a k-dimensional vector of i's assignment, and X_{-i} – a matrix of allocations of agents other than i. Let $\mathcal{F} \subset \times_{X \in \mathcal{X}} \times_{j \in K} \mathcal{F}_j(x_j) \subset R^k$ denote the set of all possible productivity profiles.

We denote an *environment* as (F, C), a set of parameters characterizing the teams' productivities and the agents' private costs on each project. Let $(\mathcal{F}, \mathcal{C})$ denote the set of possible environments.

Given an environment (F, C), the net productivity, or the *social surplus*, of a feasible allocation X equals the difference between the gross productivity and the agents' costs:

$$S(X) \equiv F(X) - \sum_{i=1}^{n} \sum_{j=1}^{k} c_{ij} x_{ij} \ . \tag{1}$$

A feasible allocation $X^* \in \mathcal{X}$ is called *efficient* if it maximizes the social surplus among all the feasible allocations. We assume that $(\mathcal{F}, \mathcal{C})$ is such that for every $F \in \mathcal{F}$ there exist $C, C' \in \mathcal{C}$ such that $S(X; C) < 0$ for every $X \in \mathcal{X} \setminus 0$ and $S(X; C') > 0$ for some $X \in \mathcal{X}$, i.e., there exist environments where production is efficient and other environments where the only efficient allocative option is "no production" $X = 0$.

Assume that the teams' productivities on each project are common knowledge, whereas the agents' costs are the agents' private information[3]. Suppose that the principal has no specific probability assessment about the distributions of the agents' costs and assumes that the agents have no common priors. Therefore, the principal is restricted to consideration of dominant strategy mechanisms[4].

[3] Almost equivalently, we can assume that the team ~~productivities ex ante are only~~ known to members of the teams, but the output of each team is ex-post observable. Then, with no uncertainty involved, a simple forcing contract (as in Holmstrom (1982)) could enforce truthful revelation of productivities as a Nash equilibrium. A coordination problem, however, prevents making the truthful productivity reports dominant strategies.

[4] If the principal were uninformed about the distribution of the agents' costs but the distribution was the common knowledge among the agents, there might exist extended revelation mechanisms in which the agents first report their common priors to the principal, and then the actual costs are revealed. In this case Bayesian equilibrium mechanisms could be considered by the principal.

The principal's problem is to offer the agents an allocation rule $X(\cdot)$ and a menu of wages $W(\cdot)$, with w_{ij} denoting the wage paid to agent i if he is employed on the project j, so that it will be always in the agents' self-interest to submit to the principal the information which will allow the latter to choose her most desirable allocation, i.e., dominant strategy incentive compatibility will be sustained. The principal seeks to maximize her profit, defined as her share of surplus net of the payments to the agents, which is

$$\pi(X) = F(X) - \sum_{i=1}^{n}\sum_{j=1}^{k} w_{ij}x_{ij} \ . \tag{2}$$

Suppose that the agents are indifferent to the outcome of production per se and care only about their own costs and wages. Specifically, we assume each agent is characterized by a quasi-linear utility function

$$u_i(x_i, w_i; c_i) = \sum_{j=1}^{k}(w_{ij} - c_{ij})x_{ij} \ . \tag{3}$$

Thus, each agent is maximizing his payoff from employment, which, given his assignment, equals the difference between his wage and cost.

By the revelation principle (Dagusta, Hammond and Maskin (1979)), without loss of generality, we can restrict our attention to direct revelation mechanisms, where the agents report their cost vectors to the principal. For each $i \in N$, let $\tilde{c}_i$ denote the reported costs as opposed to the true costs c_i. Given the reported costs $\tilde{C}$, the principal chooses allocation and wage matrices according to a prespecified rule $g(\tilde{C}) = (X(\tilde{C}), W(\tilde{C}))$[5]. The principal's task, then, is to choose a dominant strategy incentive compatible decision rule $g(\tilde{C})$ that will maximize her objective function 2. We also assume that the agents cannot be forced to participate in the project, and have a reservation utility level of 0 if they do not participate. Hence the principal has to observe the individual rationality constraints for every agent to be able to employ him. Given the above assumptions, we can present the principal's problem as follows:

$$\max_{X(\tilde{C}),W(\tilde{C})} F(X(\tilde{C})) - \sum_{i=1}^{n}\sum_{j=1}^{k} w_{ij}(\tilde{C})x_{ij}(\tilde{C}) \tag{4}$$

subject to:

$$x_{ij}(\tilde{C}) \in \{0,1\} \quad \text{for any } i \in N,\ j \in K \tag{5}$$

[5]Since the productivity environment F is observable to the principal, the rule may and will, in general, depend on F as well as $\tilde{C}$: $g = g_F(\tilde{C})$. The dependence of mechanisms on the observable elements of the environment is omitted, where possible, to simplify exposition.

$$\sum_{j=1}^{k} x_{ij}(\tilde{C}) \leq 1 \quad \text{for every } i \in N \tag{6}$$

$$\sum_{j=1}^{k}(w_{ij}(\tilde{C}_{-i}, c_i) - c_{ij})x_{ij}(\tilde{C}_{-i}, c_i) \geq \sum_{j=1}^{k}(w_{ij}(\tilde{C}) - c_{ij})\tilde{x}_{ij}(\tilde{C})$$
$$\text{for every } i \in N, \text{ any } c_i, \text{ any } \tilde{C} \tag{7}$$

$$\sum_{j=1}^{k}(w_{ij}(\tilde{C}_{-i}, c_i) - c_{ij})x_{ij}(\tilde{C}_{-i}, c_i) \geq 0$$
$$\text{for every } i \in N, \text{ any } \tilde{C}_{-i} \tag{8}$$

In the above formulation, 5 and 6 are feasibility constraints, 7 is the incentive compatibility constraint which guarantees that each agent cannot gain from a non-truthful report no matter what the others' reports are, and 8 is the individual rationality, or the voluntary participation, constraint.

The problem would be a variant of a traditional resource allocation problem if the principal were a social surplus (expression 1) maximizer; it becomes quite different when the principal's objective function is to maximize her own share of surplus (expression 2). Below, we consider the mechanisms that are feasible and optimal for a self-interested principal under various productivity-cost environments.

2.2 Complete information solution

We start with the compete information solution. Consider the payoff that would be available to the principal under complete information (principal's first best). If each agent's type were observable to the principal, she would choose the allocation and payment schedule (X^*, W^*) such that

$$\{X^*\} \text{ maximizes } F(X) - \sum_{i=1}^{n}\sum_{j=1}^{k} c_{ij}x_{ij} \ , \tag{9}$$

$$w_{ij}^* = c_{ij}x_{ij}^* \text{ for all } i \in N, \ j \in K \ . \tag{10}$$

Thus, under complete information, the principal chooses an efficient allocation – one that maximizes the social surplus. Each employed agent is compensated for the cost he bears at the project he is assigned to, and hence the individual rationality constraint is satisfied. However, the agents get a zero share of the social surplus, which goes exclusively to the principal. Note also that none of the agents can gain from changing his assignment to a different project under the suggested payment scheme since he does not get any compensation for his costs elsewhere.

In what follows, we compare the principal's payoffs under "complete ignorance" to her first best payoff and determine the information rents that the agents are able to extract from the principal.

2.3 The principal's choice of mechanisms under complete ignorance

We begin our consideration with the direct revelation dominant strategy incentive compatible (DSIC) individually rational (IR) mechanisms that are not environment-specific. Suppose the principal ex-ante has no well-defined beliefs about the distribution of the agents' costs, except, perhaps, it is known that each particular $C \in \mathcal{C}$ occurs with probability zero. We will call this a "complete ignorance" situation, keeping in mind the inaccuracy of the term. A number of possible optimality criteria can be used for comparison of various dominant strategy mechanisms under complete ignorance. For a broad class of problems with a social surplus maximizing principal, it has been shown (Groves (1973), Green and Laffont (1977), Walker (1980)) that there are DSIC mechanisms that are ex-post efficient even under the complete ignorance assumption. For any environment, they guarantee a level of social surplus no less than any other DSIC mechanisms. In the analogy with the efficiency-maximization case, we first consider the strongest possible criterion of optimality for a self-interested principal – ex-post profitability. We then discuss an alternative – and much weaker – optimality criterion.

Given the agents' cost reports $\tilde{C}$, let $(F, \tilde{C})$ denote the *reported environment.*

Definition 1 *Within the class of direct revelation DSIC IR mechanisms, a mechanism $g(C) = (X(C), W(C))^6$ is called strongly optimal if for any other DSIC IR mechanism $\tilde{g}(C) = (\tilde{X}(C), \tilde{W}(C))$, for every environment (F, C)*

$$\pi(g(C)) \geq \pi(\tilde{g}(C)) \ .$$

The following proposition shows that this optimality criterion is too strong a requirement for any incomplete information structure.

Proposition 1 *If a principal has incomplete information about the environment, there does not exist a strongly optimal DSIC IR mechanism.*

We prove the above statement with the help of several lemmas[7].

[6]Hereafter, we will denote DSIC mechanisms by $g(C)$ instead of $g(\tilde{C})$ so as to not cause confusion.
[7]The proofs for the statements, if not presented in the text, are given in section 6.

Lemma 1 *A DSIC IR mechanism is strongly optimal only if in any environment it guarantees the first best, i.e., the complete information level of profit to the principal.*

Corollary 1 *A mechanism $G(F, C)$ is strongly optimal only if it is social surplus maximizing.*

Therefore, we need to consider a class of DSIC mechanisms that maximize social efficiency. For this class of mechanisms, Green and Laffont (1977) have shown that the only truth-dominant direct revelation mechanisms are Groves mechanisms, and, moreover, these mechanisms are not generically budget balancing (Walker (1980)). We now define a class of Groves mechanisms corresponding to the team formation problem.

Definition 2 *A mechanism is called a Modified Groves mechanism if, given a reported environment $(F, \tilde{C})$, it chooses an allocation $X^*(F, \tilde{C})$ such that*

$$X^* \text{ maximizes } F(X) - \sum_{i=1}^{n}\sum_{j=1}^{k} \tilde{c}_{ij}x_{ij} \tag{11}$$

and a set of transfers $W^*(F, \tilde{C})$ defined by

$$\sum_j w^*_{ij}x^*_{ij} = F(X^*) - \sum_{l \neq i}\sum_j \tilde{c}_{lj}x^*_{lj}(\tilde{c}_i, \tilde{C}_{-i}) + h(\tilde{C}_{-i}) \ , \tag{12}$$

where $h(\tilde{C}_{-i})$ is an arbitrary deterministic function of the other agents' cost reports.

Lemma 2 *The Modified Groves mechanisms in the problem with observable production have the same properties as the standard Groves mechanisms in the allocation problem without production. That is, the Modified Groves mechanisms are the only DSIC efficient mechanisms, and they are not generically budget-balancing.*

Corollary 2 *There does not exist an efficient DSIC IR mechanism which in every environment allocates the whole social surplus to the principal.* •

Proof Follows from the fact that the Groves mechanisms are generically not budget-balancing (Walker (1980)). □

Combining the results of lemmas 1,2 and corollaries 1,2 concludes the proof of proposition 1.

Proposition 1 shows that if a self-interested principal has no (or incomplete) information about the agents' cost types, she cannot choose a mechanism that will perform better for her in any environment compared to other DSIC mechanisms. This conclusion contrasts with the results obtained for a social surplus maximizing principal: in the latter case, the principal ex-ante need not have any information about the agents' costs to implement a socially efficient outcome. The difference in the results apparently emerges from the fact that with social efficiency maximization there is "enough" coincidence of interests between the social planner and the agents; the Groves-type transfer rules compensate individuals for the differences in the social and individual objective functions. On the contrary, with profit maximization, the principal and the agents have opposing interests with respect to the social surplus division, which makes the principal's first best not implementable in dominant strategies[8].

The above results rest heavily on the optimality criterion used and the complete ignorance assumption. In section 2.5 below we consider a much weaker optimality criterion suggested by Arrow and Hurwicz (1972) for decision-making under complete ignorance; we find that under this criterion optimal mechanisms often exist. Then in section 3 we show that if the principal has a prior over the distribution of the agents' cost types, then the ex-ante optimal mechanism that maximizes the principal's expected profit subject to dominant strategy incentive compatibility is well defined. Before turning to these issues, however, we characterize certain suboptimal feasible mechanisms. In the next section, we consider efficiency-maximizing mechanisms and analyze the range of payoffs (or the share of social surplus) that the principal can guarantee for herself under these mechanisms depending on the type of environment she operates in.

2.4 Efficient dominant strategy mechanisms

Makowski and Ostroy (1987) consider the connection between the properties of efficiency-maximizing DSIC mechanisms and the competitive characteristics of an economy for a general class of incentive problems with incomplete information. They establish a direct connection between the DSIC property of mechanisms in a mechanism design framework and the notion of perfect competition in Walrasian equilibrium theory. They find that a perfectly competitive economy in which no individual can change equilibrium prices is equivalent to the special kind of DSIC IR mechanism – the marginal product mechanism under which each agent is rewarded with the level of utility exactly equal to the value of his marginal product, when agents' characteristics exhibit no complementarity with each other. Our findings presented in this section are remarkably coherent with the Makowski

[8]Roberts (1979) presents a complete characterization of the social choice functions that are implementable in dominant strategies for the class of quasi-linear utility functions. He shows that such social choice functions maximize the weighted sum of individual utilities of an allocation plus a function that does not depend on individuals' preferences. Hence, coincidence of interests between the social choice function and individual preferences is a necessary condition for implementation in dominant strategies.

and Ostroy results, although we approach the problem from a different perspective. We find that the "best" profit-maximizing mechanism for the principal restricted to the use of ex-post efficient DSIC IR mechanisms is the marginal product mechanism, and then investigate under what classes of environments the principal can, using this mechanism, extract all the social surplus from the agents.

Suppose the principal – for some reason – can only use DSIC mechanisms that are social surplus maximizing. Consider the implications of this restriction for the principal's profit. From the previous section we know (lemma 2) that the principal in this case is restricted to the class of Modified Groves mechanisms. Within this class, and taking into account that a mechanism should satisfy individual rationality, define the principal's preference over mechanisms by

Definition 3 *Within the class of Modified Groves individually rational mechanisms, a mechanism $g(C) = (X(C), W(C))$ is preferred to a mechanism $\tilde{g}(C) = (\tilde{X}(C), \tilde{W}(C))$ if for all environments (F, C)*

$$\pi(g(C)) \geq \pi(\tilde{g}(C)) \ .$$

A mechanism is called dominant if it is preferred to every other Modified Groves IR mechanisms.

Corollary 3 *A Modified Groves IR mechanism $g(C) = (X(C), W(C))$ is preferred to a Modified Groves IR mechanism $\tilde{g}(C) = (\tilde{X}(C), \tilde{W}(C))$ if and only if for every (F, C)*

$$\sum_{i \in N} h_i(C_{-i}) * (\sum_j x^*_{ij}) \leq \sum_{i \in N} \tilde{h}_i(C_{-i}) * (\sum_j x^*_{ij}) \ , \tag{13}$$

where $h_i(C_{-i})$, $\tilde{h}_i(C_{-i})$ are arbitrarily components of transfers in $g(C)$ and $\tilde{g}(C)$, respectively, as given by 12.

In the above definition the preference relation is not strict: There may exist more than one dominant Modified Groves IR mechanism. What matters, however, is that all dominant mechanisms are ex-post profit-equivalent, i.e., they provide the principal with an equal amount of profit for every environment. Therefore, it is sufficient to find just one dominant mechanism. We now introduce a Modified Groves individually rational mechanism that satisfies the desired dominance property.

Definition 4 *A direct revelation mechanism $g^*(F, \tilde{C})$ is called the Marginal Product Wage (MPW) mechanism if, given a reported environment $(F, \tilde{C})$, it chooses*

$(X^*(F,\tilde{C}), W^*(F,\tilde{C}))$ *such that*

$$X^* \text{ maximizes } F(X) - \sum_{i=1}^{n}\sum_{j=1}^{k} \tilde{c}_{ij}x_{ij} \ ; \tag{14}$$

W^* *such that for each* i

$$w^*_{ij} = \begin{cases} [F(X) - \sum_{l\neq i}\sum_j \tilde{c}_{lj}x^*_{lj}] - [F(\tilde{X}_{-i}) - \sum_{l\neq i}\sum_j \tilde{c}_{lj}\tilde{x}_{lj}] \\ \quad if \quad x^*_{ij} = 1 \\ 0 \quad if \quad x^*_{ij} = 0 \ , \end{cases} \tag{15}$$

where $\tilde{X}_{-i}$ *is an* $(n-1)\times k$ *allocation matrix that maximizes*

$$F(X_{-i}) - \sum_{l\neq i}\sum_{j} \tilde{c}_{ij}x_{ij} \equiv S(X_{-i}) \ .$$

Proposition 2 *The Marginal Product Wage mechanism is efficient, dominant strategy incentive compatible and individually rational.*

It can be easily seen that the MPW mechanism pays every employed agent his "raw marginal product" – the net marginal product that the agent produces in the most efficient allocation and his cost compensation. Let $S^* \equiv S(X^*)$ denote the social surplus produced in the efficient allocation, and $\tilde{S}_{-i} \equiv S(\tilde{X}_{-i})$ denote the social surplus produced in the efficient allocation without agent i. Then

$$\sum_j w^*_{ij}x^*_{ij} = S^* - \tilde{S}_{-i} + \sum_j \tilde{c}_{ij}x^*_{ij} \ .$$ [9]

Two important properties of the mechanism (DSIC and IR) follow: first, the agents can only gain from truthful revelation since they are rewarded with the value of the whole social surplus minus a lump sum transfer. Second, since only the agents who produce non-negative marginal social surplus are employed, each agent is guaranteed to have a non-negative level of utility. We also note that this mechanism is envy-free, i.e., no agent could gain from changing his employment given the wages he is offered[10]. The next proposition shows that the MPW mechanism is indeed dominant.

[9] It follows that each agent's utility equals the value of his (net) marginal product, and therefore this mechanism is indeed the Makowski-Ostroy marginal product mechanism. So proposition 2 directly follows from Makowski-Ostroy (1987).

[10] Formally, we call a mechanism $(X^*(\tilde{C}), W^*(\tilde{C}))$ envy-free if for every $i \in N$,

$$\sum_{j\in K}(w^*_{ij} - c_{ij})x^*_{ij} \geq \sum_{j\in K}(w^*_{ij} - c_{ij})x_{ij}$$

for any feasible x_i. Since under the MPW mechanism an agent, if employed on a project j, can only gain from employment, and is offered zero wages at the projects other than j, the envy-free requirement is satisfied.

Proposition 3 *The Marginal Product Wage mechanism is dominant in the class of Modified Groves individually rational mechanisms.*

Knowing that the Marginal Product Wage mechanism is "the best" for the principal in the class of efficient DSIC IR mechanisms, we turn to the question of how profitable this mechanism could be. Unfortunately, as the next proposition shows, the principal is not guaranteed against losses under this mechanism.

Proposition 4 *The principal cannot guarantee herself a non-negative profit for every environment under the Marginal Product Wage mechanism.*

Proof It is sufficient to present an example of an environment in which the principal gets a negative payoff. Let $n = 2$, $k = 2$, $F_1(\{1\}) = 5$, $F_1(\{2\}) = 7$, $F_1(\{1,2\}) = 15$, $F_2(\{1\}) = 0$, $F_2(\{2\}) = 3$, $F_2(\{1,2\}) = 4$, $c_{ij} = 2$ for all i, j. Then the efficient allocation is $x^*_{11} = 1$, $x^*_{12} = 0$, $x^*_{21} = 1$, $x^*_{22} = 0$, with $F(X^*) = 15$. The MPW mechanism wages are $w_{11} = (15-2) - (7-2) = 8$, $w_{21} = (15-2) - (5-2) = 10$, $w_{12} = w_{22} = 0$. Then the principal's payoff is $\pi(X^*) = 15 - 8 - 10 = -3 < 0$. □

Under specific types of environments, however, the principal is able to extract a non-negative surplus from the agents. These are highly competitive environments, in which the agents' marginal contributions to the total surplus produced are "low enough." We now characterize these environments.

Corollary 4 *If the agents' net marginal products $(S^* - \tilde{S}_{-i})$ are "low enough" in the sense that the following inequality holds:*

$$(n-1)S^* \leq \sum_{i=1}^{n} \tilde{S}_{-i} \ , \tag{16}$$

then the principal gets a non-negative payoff in the MPW mechanism.

The above conditions may correspond to different types of economic situations: in one type, an efficient allocation does not employ every available agent, but for many employed agents there are unemployed ones that closely match them in productivity and cost characteristics. In this case there exists a high degree of substitution among some agents, and an environment can be called highly competitive. The other possible situation is when most agents are employed but the teams' net productivities are characterized by "decreasing returns to scale." We start with analysis of the latter case. Consider an environment in which it is efficient to employ every available agent under the MPW mechanism. We call it a *full employment* environment.

Definition 5 *A full employment environment is one in which every available agent is employed under the MPW mechanism. Formally, given the set of agents N and any $N_h \subseteq N$, let $X^*(N_h)$ be the matrix of efficient assignments when the subset N_h of agents is available. Then the environment is called full employment if*

$$\sum_{i \in N_h} \sum_j x^*_{ij}(N_h) = |N_h| \text{ for all } N_h \subseteq N \ , \tag{17}$$

where $|N_h|$ denotes the number of elements in N_h.

Next, we introduce the notion of decreasing returns to scale. For our model it is easier to define decreasing returns to scale in terms of average per person surplus (net product) produced under the MPW mechanism. Let $s^*(N) = S^*/(\sum_i \sum_j x^*_{ij})$ denote the average per person surplus produced when all the agents are available for employment, and $\tilde{s}_{-i} = \tilde{S}(N_{-i})/(\sum_{l \neq i} \sum_j \tilde{x}_{lj})$ – average per person surplus when all the agents but i are available. Finally, let $\bar{s}(N_{-i}) = (\sum_{i=1}^n \tilde{s}_{-i})/n$ denote the average per person surplus produced when an "average" agent is excluded from possible employment.

Definition 6 *A production environment is characterized by decreasing net returns to scale if an average agent, efficiently employed under MPW mechanism, decreases the average per person social surplus produced, as compared to the efficient employment without this agent. That is,*

$$s^*(N) \leq \bar{s}(N_{-i}) \ . \tag{18}$$

Corollary 5 *If an environment is full employment and characterized by decreasing net returns to scale, the principal can guarantee herself a non-negative payoff under the MPW mechanism.*

We now turn to another type of environment which is extremely favorable to the principal – a perfectly competitive environment, where no agent is indispensable. In such environments, each agent faces the competition of at least one other agent who is identical to him. With a continuum of possible types of agents and independence of types, such environments occur with probability zero, but if the number of agents available for employment is large, there might exist ~~some agents who~~ closely match each other in productivity-costs characteristics. Then each agent's marginal contribution, and, consequently, his share of the social surplus will be small, therefore increasing the principal's share of the surplus. The analysis of the extreme, perfect competition case indicates that, in general, competition serves the interests of the principal.

Definition 7 *An environment (F, C) is called perfectly competitive if for any agent $i \in N$ who is employed under MPW mechanism*

$$S(X^*(F, C)) = S(\tilde{X}_{-i}(F, C_{-i}))^{11} \ .$$

Proposition 5 *If an environment is perfectly competitive, then under the MPW mechanism the principal achieves her first best level of profit, i.e., she captures the whole social surplus of production.*

This statement follows from the definitions of the MPW mechanism and the perfectly competitive environment.

To summarize, we find that efficient dominant strategy individually rational mechanisms do not always leave the principal with a non-negative profit. The environments in which the principal can guarantee herself a non-negative profit are rather restrictive and look a lot like traditional labor markets with homogeneous workers; competition among agents serves to the principal's advantage. On the other hand, if the agents have complementary characteristics and their joint production in teams produces increasing social surplus, the principal is doomed to run into losses. The last observation is curious since from a production efficiency viewpoint, the teams should be formed exactly when the efficiency gain from the joint production is high. Our analysis indicates, however, that a self-interested principal who is forced to implement efficient allocations only loses from high efficiency.

2.5 An alternative optimality criterion

The above analysis does not imply, of course, that any DSIC IR mechanism would necessarily make a self-interested principal bear losses under certain environments. It only shows that no-loss DSIC mechanisms cannot have socially desirable properties such as economic efficiency. From the profit-maximizing principal's perspective, however, efficiency is not nearly as important as the profits that a mechanism produces in every possible environment. In section 2.3 above we have found that there is no DSIC IR mechanism that is ex-post profit maximizing for all environments. With this result in mind, the principal may prefer any mechanism which insures her against losses and provides high profits at least in some environments. This reasoning corresponds to the criterion of optimality suggested by Arrow and Hurwicz (1972) for decision-making under ignorance. In their terms, an action is called optimal if the minimal and maximal

[11]Makowski and Ostroy define perfect competition as a situation in which no individual can change equilibrium prices. They further show that this is equivalent to an environment where each agent's marginal product equals zero.

possible payoffs from this action are not lower than the respective payoffs from any other action. In application to our problem, this criterion implies that a mechanism is optimal if it always provides a non-negative profit and in the environment with minimal costs guarantees the principal her first best[12]:

Definition 8 *Within the class of direct revelation DSIC IR mechanisms, a mechanism* $g(C) = (X(C), W(C))$ *is called weakly optimal if for any* $C \in \mathcal{C}$

$$\pi(g(C)) \geq 0$$

and there exists $C^* \in \mathcal{C}$ *such that for any* $C \in \mathcal{C}$*, any* $X \in \mathcal{X}$

$$\pi(g(C^*)) \geq S(X; C) \ .$$

From the above discussion, it immediately follows that if $\mathcal{C}$ is closed from below, then a weakly optimal mechanisms exists.

Proposition 6 *Suppose that* $C^* \in \mathcal{C}$*, where* C^* *is defined by* $c^*_{ij} \equiv \inf\{c_{ij} | c_{ij} \in C_{ij}\}$ *for all* $(i,j) \in N \times K$*. Then there exists a weakly optimal mechanism.*

Proof Suppose that $C^* \in \mathcal{C}$, where C^* is defined as above. Consider a mechanism $g^*(C)$ which chooses an efficient allocation X^* when $C = C^*$ is reported, and no production $X^* = 0$ otherwise. Let $w^*_{ij} = c_{ij} x^*_{ij}$. Obviously, g^* is weakly optimal. □

Unfortunately for the principal, weakly optimal mechanisms may always result in zero profit except for a single lowest cost environment. We now suggest an alternative suboptimal class of "no-loss" mechanisms which allow the possibility of positive profits for uncountable sets of environments.

Consider the following direct revelation mechanism. Given the observable productivity parameters $F(\cdot)$, choose an arbitrary $n \times m$ matrix of constants B, $B \in \mathcal{C}$, such that

$$\max_{X \in \mathcal{X}} (F(X) - \sum_{i \in N} \sum_{j \in K} b_{ij} x_{ij}) > O \ .$$

[12]Note that, first, there do not exist individually rational mechanisms that guarantee strictly positive profits in all environments, since we have assumed (section 2.1) that there is $C \in \mathcal{C}$ such that for any feasible X, $S(X) \leq 0$. For the same reason, any mechanism that gives the principal a profit higher than the corresponding level of the social surplus cannot be individually rational.

Let $X^*(B)$ denote an allocation where the maximum is achieved. Next, given the cost reports $\tilde{C}$, form the matrix $\tilde{X}(F,\tilde{C})$ using the following rule:

$$\tilde{x}_{ij} = \begin{bmatrix} 1 & \text{if } x^*_{ij}(B) = 1 \text{ and } \tilde{c}_{ij} \leq b_{ij} \\ 0 & \text{otherwise .} \end{bmatrix}$$

Now consider the mechanism $\hat{g}(\tilde{C}) = (\hat{X}(\tilde{C}), \hat{W}(\tilde{C}))$ such that $\hat{X}$ solves

$$\begin{aligned} &\max_X (F(X) - \sum_{i\in N}\sum_{j\in K} b_{ij}x_{ij}) \\ &\text{subject to:} \\ &X \text{ is feasible },\\ &F(X) - \sum_{i\in N}\sum_{j\in K} b_{ij}x_{ij} \geq 0 \\ &\text{if } \tilde{x}_{ij} = 0 \text{ then } \hat{x}_{ij} = 0 \ , \end{aligned}$$

and $\hat{W}$ is defined by

$$\hat{w}_{ij} = \begin{bmatrix} b_{ij} & \text{if } \hat{x}_{ij} = 1 \\ 0 & \text{otherwise .} \end{bmatrix}$$

This mechanism is DSIC, IR and never gives the principal a negative profit. For $C^* = B$, the mechanism provides the principal her first-best outcome; for any $C \in \mathcal{C}$ such that $c_{ij} \leq b_{ij}$ for all $(i,j) \in N \times K$, the principal gets a positive profit. In this respect, such suboptimal mechanisms may be more reasonable than the weakly optimal ones. Yet, these mechanisms are still generically inefficient and not profit-maximizing, with the resulting allocations being chosen almost ad hoc; the only purpose of the agent's cost reports may be to insure individual rationality. The "constant rule" weakly optimal mechanisms which always assign the same allocation and wages, unless vetoed by the agents, are even less profitable and sensitive to the environment then the mechanism suggested above. Unfortunately, it is hard to find a no-loss dominant strategy IR mechanism that produces an allocation which is responsive to the cost reports without disturbing the agents' incentives to report the truth. For example, suppose the principal adapts an MPW mechanism truncated at zero level of profit: given the reported environment, she uses the MPW mechanism if it gives her non-negative profit, and chooses not to engage the production process otherwise. Then the agents might be tempted to misrepresent their costs in favor of less efficient allocations for fear of having no production in the case of truthful reports.

To conclude, we find that if the principal has no information about the agents' costs, in the sense specified earlier, the dominant strategy incentive compatible mechanisms that she might use are either almost ad hoc and not sensitive to the agent's private information, or make the principal bear losses in some environments. There is no strongly optimal mechanism for the principal. The result is not surprising in view of the principal's lack of information and a strong dominant strategy requirement imposed on the agent's behavior. We next consider how our conclusions change if we move away from the principal's complete ignorance assumption.

In the next section, we consider the Bayesian Nash equilibrium framework and characterize optimal Bayesian mechanisms for the expected profit-maximizing principal. We then return to the question of their dominant strategy implementation.

3 Expected profit maximizing mechanisms

The results obtained for dominant strategy mechanisms under complete ignorance do not require any assumptions on the consistency of agents' costs characteristics across projects. In this section we introduce such assumption to be able to reduce the general problem to a special single-dimensional case. Perhaps surprisingly, we find that the conditions necessary for Bayesian incentive compatibility of team-formation mechanisms can be quite restrictive even under this strong assumption.

3.1 The model

Suppose now that the principal, when hiring an agent, can observe his profession, but cannot recognize the quality of the training, or the agent's type. In the other words, the principal can tell an engineer from a carpenter, but does not know how qualified each of them is. In general, an agent's qualification (type) may affect both the team's output and the agent's personal costs. Yet in what follows we assume that the team's output is a deterministic function of the agents' professions and is uninformative about their quality[13]. The role of agents' types is to effect their personal costs of performing any task in a consistent way. The high-quality agents bear lower costs compared to the low-quality agents of the same profession. The problem of type revelation exists because of the presence of individual rationality constraints.

Formally, assume that for every agent i, $i \in N$, his cost of working on each project j, $j \in K$, is a deterministic twice continuously differentiable non-increasing function of his single-dimensional type t_i, $c_{ij} = c_{ij}(t_i)$, $c'_{ij}(t_i) \leq 0$, $c''_{ij}(t_i) \geq 0$. The agents' types are stochastically independent random variables, distributed over the supports $T_i = [0, \bar{t}_i] \subset R$ according to the probability distributions $H_i(t_i)$. Each $H_i(t_i)$ is twice continuously differentiable, with corresponding density $h_i(t_i)$, $i \in N$. The distribution functions conform to the Monotone Hazard Rate property, i.e., for every $i \in N$,

[13]For example, we can imagine that a forcing contract makes each team produce an assigned task.

$(1 - H_i(t_i))/h_i(t_i)$ is non-increasing in t_i[14]. Let $H(t)$ and $h(t)$ denote the cumulative distribution and density functions of vectors of types $t = (t_1, .., t_n)$ over the support $T = \times_i T_i$, and $H_{-i}(t_{-i})$, $h_{-i}(t_{-i})$ denote the corresponding distribution functions of vectors $t_{-i} = (t_1, .., t_{i-1}, t_{i+1}, .., t_n)$. For all $i \in N$, the support T_i, the probability distribution function $H_i(t_i)$, and the cost functions $c_i(\cdot) : R \to R^k$ are common knowledge. Each agent's cost type is his private information. Both the principal and the agents are risk-neutral. The agents follow Bayesian Nash Equilibrium behavior. The principal's purpose is to construct a Bayesian incentive compatible (BIC) IR mechanism that maximizes her expected profit. Using the revelation principle[15], we again restrict our attention to the direct revelation mechanisms. Consider mechanisms that determine probabilities of matrix allocations and wages for the agents as a function of their reported types $\tilde{t}$.

Let $\mathcal{X} = \{X | X \, is \, feasible\}$ denote the set of all feasible matrix allocations. We can arbitrarily order the elements in $\mathcal{X}$ so that $\mathcal{X} = \{X_1, .., X_l, .., X_{(k+1)^n}\}$. Let $L = \{1, .., l, .., (k+1)^n\}$ be the set of corresponding indices, and for every $l \in L$ let $F_l \equiv F(X_l)$. Then we can consider a $(k+1)^n$-dimensional probability vector $P = \{p_1, .., p_l, .., p_{(k+1)^n}\}$, where p_l represents the probability of l-th feasible matrix allocation X_l. This implies the new feasibility conditions:

$$p_l \geq 0 \quad \text{for all } l \in L$$
$$\sum_{l \in L} p_l = 1 \ .$$

Equality in the second condition above indicates that $X = 0$ is a feasible allocation.

Further, let $W = (w_1, .., w_i, .., w_n)$ denote the vector of agents' wages. A direct revelation mechanism then is a function from the reported types $\tilde{t} \in T$ into the probability vector $P \in R^{(k+1)^n}$ and the wage vector $W \in R^n$: $g(\tilde{t}) = (P, W)$. We restrict our attention to the mechanisms such that $P(t)$ is piecewise continuously differentiable.

Some additional notation is needed for further analysis. For a given probability vector P, for every $i \in N$, $j \in K$, let $L_{ij}(P) \subset L$ denote the set of indices whose corresponding allocations assign agent i to project j. Then

$$q_{ij}(P) = \sum_{l \in L_{ij}(P)} p_l$$

denotes the probability of agent i being assigned to project j. Given the profile of reported

[14]The Monotone Hazard Rate property together with $c''_{ij}(t_i) \geq 0$ are the standard assumptions made in the literature to guarantee that monotonicity constraints (see below) are not binding in the optimal allocation problems with single-dimensional decision space (see, for example, Fudenberg and Tirole (1992)). As we demonstrate in what follows, the assumptions remain important in the analysis of multi-dimensional allocation problems as well.

[15]See, for example, Myerson (1979).

strategies $\tilde{t}(t)$, agent i's utility under the mechanism $(P(\tilde{t}), W(\tilde{t}))$ is:

$$u_i(P, W, \tilde{t}, t) = w_i(\tilde{t}) - \sum_{j \in K} c_{ij}(t_i) * \sum_{l \in L_{ij}(P)} p_l(\tilde{t}) \ . \tag{19}$$

Similarly, i's expected probability of being assigned to a project j is

$$Q_{ij}(\tilde{t}, t_i) = \int_{T_{-i}} \sum_{l \in L_{ij}(P)} p_l(\tilde{t}(t_{-i}, t_i)) h_{-i}(t_{-i}) \, dt_{-i} \ , \tag{20}$$

and i's expected utility is, correspondingly,

$$\begin{aligned} &U_i(P, W, \tilde{t}, t_i) = \\ &\quad = \int_{T_{-i}} [w_i(\tilde{t}(t_{-i}, t_i)) - \sum_{j \in K} c_{ij}(t_i) * \sum_{l \in L_{ij}(P)} p_l(\tilde{t}(t_{-i}, t_i))] h_{-i}(t_{-i}) \, dt_{-i} \ . \end{aligned} \tag{21}$$

Given that the agents' type is t and their reported strategy is $\tilde{t}(\cdot)$, the principal gains the profit

$$\pi(P, W, \tilde{t}, t) = \sum_{l \in L} F_l p_l(\tilde{t}(t)) - \sum_{i \in N} w_i(\tilde{t}(t)) \tag{22}$$

and his expected profit is

$$\Pi(P, W, \tilde{t}) = \int_T (\sum_{l \in L} F_l p_l(\tilde{t}(t)) - \sum_{i \in N} w_i(\tilde{t}(t))) h(t) \, dt \ . \tag{23}$$

Let $u_i(P, W, t)$ and $U_i(P, W, t_i)$ denote agent's i utility and expected utility when the agents report their true types, i.e., $\tilde{t}(t) = t$. Then the principal's problem can be stated as follows:

$$\max_{P(t), W(t)} \int_T (\sum_{l \in L} F_l p_l(t) - \sum_{i \in N} w_i(t)) h(t) \, dt \tag{24}$$

subject to:

$$p_l(t) \geq 0 \quad \text{for any } l \in L \tag{25}$$

$$\sum_{l \in L} p_l(t) = 1 \tag{26}$$

$$\begin{aligned} &U_i(P, W, t_i) \geq \\ &\quad \geq \int_{T_{-i}} (w_i(t_{-i}, \tilde{t}_i) - \sum_{j \in K} c_{ij}(t_i) * \sum_{l \in L_{ij}(P)} p_l(t_{-i}, \tilde{t}_i)) h_{-i}(t_{-i}) \, dt_{-i} \\ &\qquad \text{for every } i \in N, \text{ any } t_i, \text{ any } \tilde{t}_i \end{aligned} \tag{27}$$

$$U_i(P, W, t_i) \geq 0 \quad \text{for every } i \in N, \text{ any } t_i \tag{28}$$

Here the feasibility constrains take the form of 25-26, and the incentive compatibility (BIC) 27 and individual rationality 28 constrains are written assuming Bayesian equilibrium behavior.

A direct revelation mechanism $(P(t), W(t))$ is *optimal* if it solves the problem 24-28.

3.2 Optimal Bayesian equilibrium mechanisms

Note that the assumption $c_i'(t_i) \leq 0$ insures that the single-crossing property, or the constant sign condition (Guesnerie, Laffont (1984)) holds; i.e., the agents' costs on each project change with their types in a consistent manner. This allows us to use the standard techniques developed for the analysis of mechanism design problems in a Bayesian framework[16]. Consider the necessary conditions for Bayesian incentive compatibility[17].

Proposition 7 *(Interim monotonicity) If a feasible direct revelation mechanism $(P(t), W(t))$ is Bayesian incentive compatible, then for any $i \in N$, any $s_i, t_i \in T_i$ the following is true:*

$$\sum_{j \in K} (c_{ij}(t_i) - c_{ij}(s_i))(Q_{ij}(P, t_i) - Q_{ij}(P, s_i)) \leq 0 \ . \tag{29}$$

We call the above condition "monotonicity" in analogy to the one-project case, where the condition reduces to the requirement that the expected probability of employment is monotonic in an agent's type. For the multi-project case the condition becomes more demanding. First, as in a one-project case, it requires that an agent of a higher type should be hired with higher expected probability than an agent of a lower type. Second, with respect to shifting an agent's employment probabilities among the projects, it requires that an agent should be more likely to be assigned to the project where his cost decrease is the fastest among the projects. We first solve for the optimal mechanism assuming that the necessary conditions for BIC hold, and then characterize the conditions under which this assumption holds.

In the spirit of Myerson's (1981) analysis, for every $i \in N$, $j \in K$, let

$$J_{ij}(t_i) \equiv c_{ij}(t_i) - c_{ij}'(t_i) \frac{1 - H_i(t_i)}{h_i(t_i)} \tag{30}$$

denote agent's i *virtual cost* of working on a project j. Similarly, define a *virtual surplus* $\tilde{S}(X)$ of an allocation X by

$$\tilde{S}(X; t) = F(X) - \sum_{i \in N} \sum_{j \in K} J_{ij}(t_i) x_{ij} \ . \tag{31}$$

[16]See Fudenberg and Tirole (1992), chapter 7, for an overview.

[17]The proofs for the propositions in this section are given in section 6.

Proposition 8 *A direct revelation mechanism* $(P(t), W(t))$ *of the form*

$$p_l = \begin{bmatrix} 1 & \text{if } X_l \text{ maximizes } \tilde{S}(X;t) \text{ subject to} \\ & \text{interim monotonicity constraint 29;} \\ 0 & \text{otherwise ;} \end{bmatrix} \tag{32}$$

$$w_i(t_i) = \sum_{j \in K} c_{ij}(t_i) Q_{ij}(t_i) - \int_0^{t_i} \sum_{j \in K} c'_{ij}(\tau_i) Q_{ij}(\tau_i)\, d\tau_i$$
$$\text{for all } i \in N, \text{ all } t_i \in T_i \tag{33}$$

is optimal for the principal[18].

Hence, an optimal mechanism, within the constraint imposed by monotonicity, chooses an allocation that maximizes virtual social surplus and offers each agent an expected payment which is never lower than his expected costs of employment (the latter follows from the assumption that $c_i'(t_i) \leq 0$). Similar to the optimal auctions results (Myerson (1981)), we find that, first, the optimal team-formation mechanisms are generically inefficient, since the principal trades off some efficiency for higher expected profit. Second, there exists the whole class of equivalent optimal BIC mechanisms which differ from each other in the form of actual wages paid to the agents; expected wages are given by 33. Also note that, similarly to the optimal auction, an agent's probability of being hired under an optimal mechanism is a non-decreasing function of his type: Provided that the Monotone Hazard rate condition and the assumption $c''_{ij}(t_i) \geq 0$ hold for all i, j, we get that $J'_{ij}(t_i) \leq 0$ for any i, j. This important observation is useful for the further analysis; we state it as the following lemma:

Lemma 3 *Under an optimal mechanism of the form 32-33, for any* $t \in T$, *any* $i \in N$, *agent* i*'s probability of employment* $\sum_{j \in K} x_{ij}(t_{-i}, t_i)$ *is a non-decreasing function of his type* t_i.

The optimal BIC mechanisms differ depending on whether the interim monotonicity constraints 29 are ever binding. If $H(t)$ and c_i's are such that the constraints are not binding, then the optimal mechanism is explicitly given in Proposition 7. Otherwise, bunching is optimal over the ranges of types where the monotonicity is binding[19]. We now proceed with the analysis of the restrictiveness of the monotonicity constraints 29.

[18] In the above statement we ignore the possibility that there may exist more than one allocation X that maximizes the virtual surplus. Generically, these cases occur with probability zero. However, if such a situation emerges, an optimal mechanism, equivalent to 32-33, randomly chooses one of the efficient allocations.

[19] See Guesnerie and Laffont (1984) for an exposition of bunching technique.

3.3 Analysis of monotonicity requirements

To see that the constraints 29 can be indeed rather restrictive and often binding, consider the following example.

Example *(Restrictiveness of the monotonicity constraint)* Consider the problem of hiring one agent on two alternative projects, i.e., let $n = 1$, $k = 2$. Let $F_1 = 100$, $F_2 = 98$. Suppose that the agent's cost type t is uniformly distributed on $[0, 5]$, and let $c_1(t) = 3/(t + 0.1) + 10$, $c_2(t) = 20 - 4t$. Note that the uniform distribution conforms to the Monotone Hazard Rate property and the cost functions are decreasing and convex, as required by the initial assumptions. We demonstrate that the monotonicity condition 29, which in this case takes the form of ex-post monotonicity,

$$\sum_{j \in K} (c_j(t) - c_j(s))(x_j(P, t) - x_j(P, s)) \leq 0 , \tag{34}$$

is not trivially satisfied. Take $s = 0.23$ and $t = 2.5$ and let us compute the virtual surplus maximizing allocations $X(s)$, $X(t)$. Since $c_1(s) = 19.08$, $c_2(s) = 19.08$, $c_1(t) = 11.15$, $c_2(t) = 10$; $(1 - H(s))/h(s) = 0.19$, $(1 - H(t))/h(t) = 0.1$; $c_1'(s) = -27.59$, $c_2'(s) = -4$, $c_1'(t) = -0.44$, $c_2'(t) = -4$, we obtain $J_1(s) = 24.7$, $J_2(s) = 19.84$, $J_1(t) = 11.55$, $J_2(t) = 10.04$. Therefore,

$$\begin{array}{llll} \tilde{S}_1(s) = 75.3 & \tilde{S}_2(s) = 78.2 & \tilde{S}_1(t) = 88.45 & \tilde{S}_2(t) = 87.96 \\ x_1(s) = 0 & x_2(s) = 1 & x_1(t) = 1 & x_2(t) = 0 . \end{array}$$

However, the above allocations do not conform to the monotonicity constraint 34 and therefore cannot be chosen:

$$\begin{aligned} &(c_1(t) - c_1(s))(x_1(t) - x_1(s)) + (c_2(t) - c_2(s))(x_2(t) - x_2(s)) = \\ &= (11.15 - 19.08)(1 - 0) + (10 - 19.08)(0 - 1) = 1.15 > 0 . \end{aligned}$$

Figure 1 gives a graphical illustration of the above example. Note that the graphs of the cost functions for the two projects intersect more than once, which indicates that there is no consistency in the cost change on one project relative to the other. Specifically, there is a switch in the relative rate of cost decrease between the projects: $c_1'(s) < c_2'(s)$, but $c_1'(t) > c_2'(t)$. As a consequence it is possible that with the change in type the true costs change in favor of one project, whereas the virtual costs change in favor of the other. Under these rather typical circumstances, the monotonicity constraint can be violated quite easily, as the example above shows. Below, we present sufficient conditions for monotonicity, which guarantee that the above situation is never the case[20]. The condition states that the difference in the rates of cost decreases between any two projects does not change "too much" compared to the difference in the change in the absolute costs between the projects.

[20]The sufficient conditions for monotonicity presented in propositions 9 and 10 are specific for $x_{ij} \in \{0, 1\}$ case.

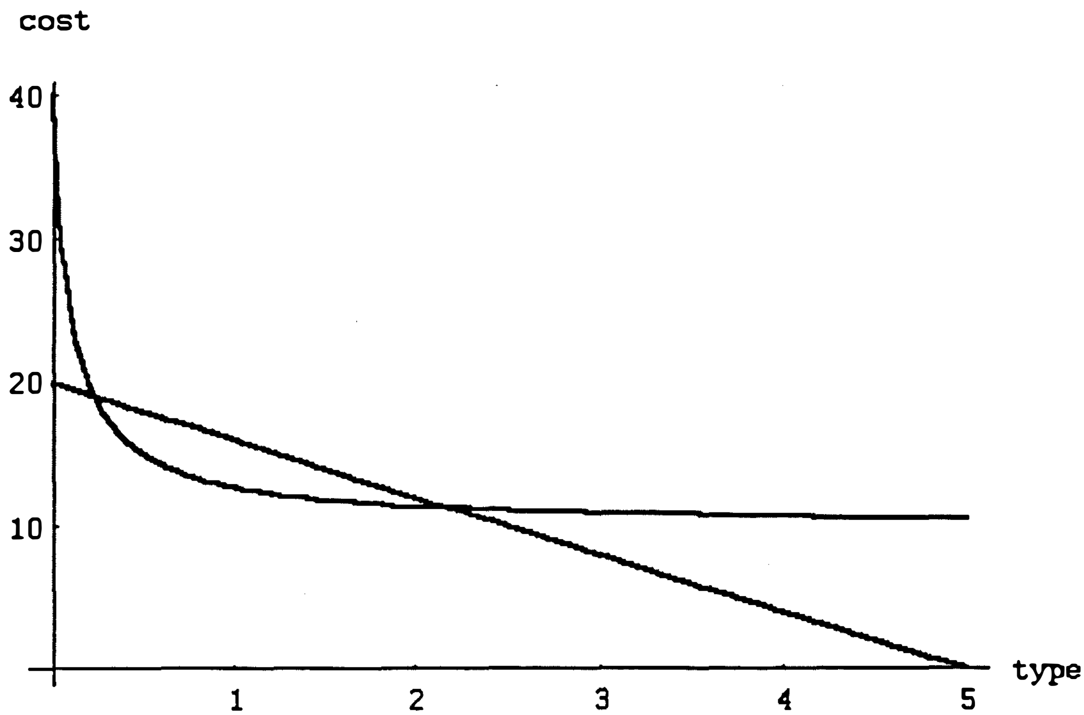

Figure 1: An example of cost functions violating sufficient conditions for monotonicity: $c_j(t) = 3/(t+0.1)+10$, $c_k(t) = 20-4t$, $t \in [0,5]$.

Proposition 9 *(Sufficient conditions for monotonicity) The monotonicity conditions 29 are satisfied if the following is true:*

For every $i \in N$, any $s_i, t_i \in T_i$, any $j, k \in K$, $j \neq k$, if

$$(c_{ij}(s_i) - c_{ij}(t_i)) - (c_{ik}(s_i) - c_{ik}(t_i)) > 0 \ , \tag{35}$$

then

$$(c_{ij}(s_i) - c_{ij}(t_i)) - (c_{ik}(s_i) - c_{ik}(t_i)) > (\frac{1-H_i(s_i)}{h_i(s_i)}c'_{ij}(s_i) - \frac{1-H_i(t_i)}{h_i(t_i)}(c'_{ij}(t_i)) - $$
$$-(\frac{1-H_i(s_i)}{h_i(s_i)}c'_{ik}(s_i) - \frac{1-H_i(t_i)}{h_i(t_i)}(c'_{ik}(t_i)) \ . \tag{36}$$

Moreover, the above condition is also sufficient to guarantee that the ex-post monotonicity requirement is satisfied: if for any $i \in N$, any $s_i, t_i \in T_i$ 35 implies 36, then for all $t_{-i} \in T_{-i}$ the following inequality holds:

$$\sum_{j \in K}(c_{ij}(t_i) - c_{ij}(s_i))(x_{ij}(t) - x_{ij}(s_i, t_{-i})) \leq 0 \ . \tag{37}$$

Outline of the proof The complete proof is given in the section 6; here we present the outline to show that the above conditions guarantee not only interim, but also ex-post monotonicity.

We show that virtual surplus maximization

$$\max_X \{F(X) - \sum_{i \in N} \sum_{j \in K} J_{ij}(t_i) x_{ij}\} \tag{38}$$

implies that for any $i \in N$, any s_i, t_i

$$\sum_{j \in K} (J_{ij}(t_i) - J_{ij}(s_i))(x_{ij}(t_i) - x_{ij}(s_i)) \leq 0 \ , \tag{39}$$

which, under the conditions stated in the proposition, in turn implies

$$\sum_{j \in K} (c_{ij}(t_i) - c_{ij}(s_i))(x_{ij}(t_i) - x_{ij}(s_i)) \leq 0 \ . \tag{40}$$

The latter inequality is ex-post monotonicity, which is clearly stronger than interim monotonicity and implies it. □

The form of the sufficient conditions together with the earlier example suggest that the conditions that guarantee monotonicity are not trivial and do not generically hold. Rather, they are satisfied for certain groups of type distributions and cost functions. In particular, the monotonicity conditions hold if each agent's costs change in a consistent manner not only with types, but also from project to project. We state this case in the following proposition.

Proposition 10 *Suppose that for every $i \in N$, for every pair of projects $j, k \in K$, either*

$$(i) \quad c'_{ij}(t_i) < c'_{ik}(t_i) \ \textit{and} \ c''_{ij}(t_i) \geq c''_{ik}(t_i) \ \textit{for all} \ t_i \in Ti \ , \tag{41}$$

or

$$(ii) \quad c'_{ij}(t_i) \geq c'_{ik}(t_i) \ \textit{and} \ c''_{ij}(t_i) \leq c''_{ik}(t_i) \ \textit{for all} \ t_i \in Ti \ . \tag{42}$$

Then the monotonicity conditions 29, as well as 37, are satisfied.

The set of assumptions presented in the proposition 10 implies that for each agent, the projects are ranked with respect to the rates in cost changes; this ranking does not change with types. This can be interpreted as a single-crossing property for the cost functions of each agent: under the given conditions, an agent's cost functions for any two projects j and k can intersect at most once (compare this to the cost functions in the example above). Figure 2 illustrates the idea.

The above versions of sufficient conditions for the monotonicity, together with the standard single-crossing property guaranteed by the assumption $c'_{ij} \leq 0$ for any i, j,

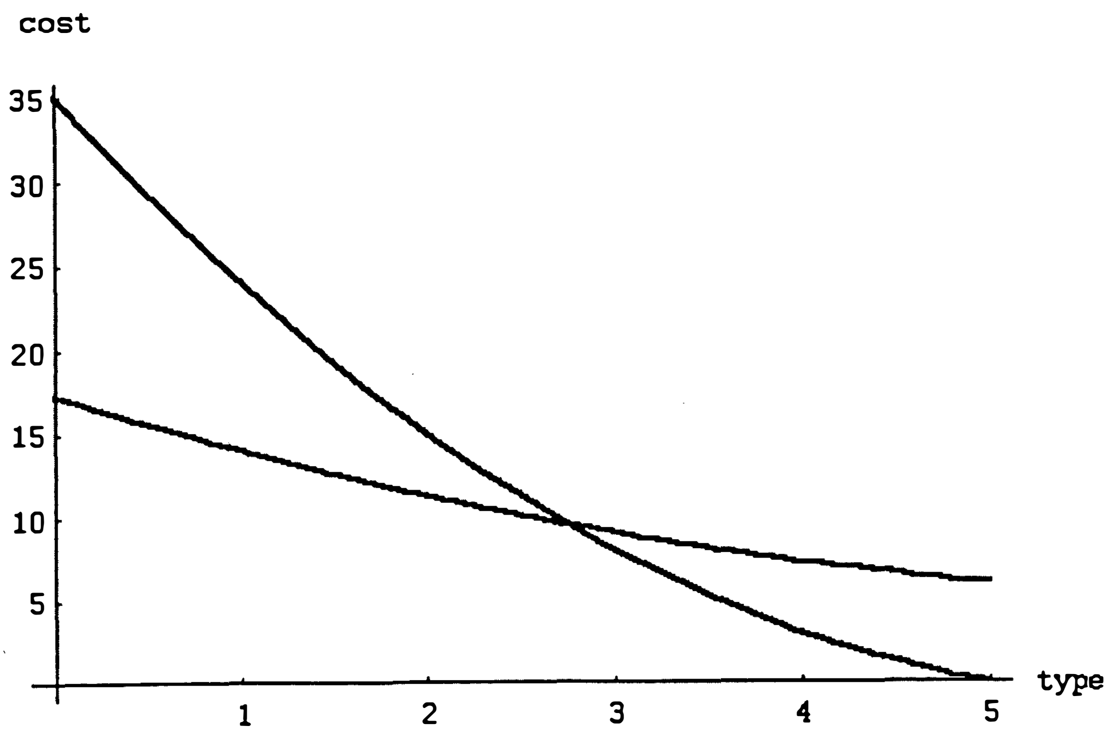

Figure 2: An example of cost functions satisfying sufficient conditions for monotonicity: $c_j(t) = (x-6)^2 - 1$, $c_k(t) = (x-7)^2/4 + 5$, $t \in [0, 5]$.

present a set of very restrictive regularity requirements. Although the necessary and sufficient conditions[21] for the interim monotonicity may be less restrictive, the example presented in this section demonstrates that some consistency in each agent's cost functions for different projects may still be required. The above analysis indicates that the monotonicity conditions become much harder to satisfy once we move from one-dimensional to multi-dimensional decision spaces. Therefore, under a broad range of circumstances, bunching will be optimal over wide ranges of an agent's type. This suggests that multidimensionality of decision variables, through making monotonicity conditions more restrictive, may substantially decrease the principal's expected profit compared to a one-dimensional (one-project) allocation problem.

For the rest of the section, we restrict our attention to the cases when the necessary monotonicity conditions are not binding, and therefore the optimal BIC mechanism is of the form explicitly presented in proposition 7.

[21]The necessary and sufficient conditions are not considered here for the reason that they differ depending on the values of the observed productivity parameters; obtaining sensible necessary conditions requires imposition of certain regularity requirements on productivities, which would narrow the scope of the analysis.

3.4 Implementation in dominant strategies

Following the results of Mookherjee and Reichelstein (1992), we now show that an optimal BIC mechanism can be equivalently implemented in dominant strategies with no expected loss to the principal. This brings us back to the problem initially stated in section 2 – the one of finding an optimal profit-maximizing dominant strategy incentive compatible mechanism. The following result shows that if the principal knows a prior distribution of the agents' cost types there exists an optimal, in the sense of expected profit maximization, DSIC IR mechanism.

Proposition 11 *Suppose that the cost functions $c_i(t_i)$ and the distribution functions $H_i(t_i)$, $i \in N$, are such that the ex-post monotonicity conditions 37 are satisfied. Then the direct revelation mechanism $(X(t), W(t))$, given by*

$$X^*(t) = X \text{ that maximizes } \tilde{S}(X;t)\ , \tag{43}$$

$$w_i(t) = \sum_{j \in K} c_{ij}(t_i) x^*_{ij}(t) - \int_0^{t_i} \sum_{j \in K} c'_{ij}(\tau) x^*_{ij}(t_{-i}, \tau)\, d\tau$$
$$\text{for all } i \in N,\ t_i \in T_i\ ,\ t_{-i} \in T_{-i}\ , \tag{44}$$

is a dominant strategy incentive compatible and ex-post individually rational mechanism which yields the same expected profit as the optimal Bayesian incentive compatible mechanism 32-33.

The above proposition says that we can replace the optimal BIC mechanism with an equivalent DSIC mechanism[22]. Moreover, since the DSIC constraints are more restrictive than the BIC constraints, we have also established

Corollary 6 *If the cost functions $c_i(t_i)$ and the distribution functions $H_i(t_i)$, $i \in N$, are such that the sufficient conditions for monotonicity (proposition 9) are satisfied, then 43-44 is an optimal expected profit maximizing dominant strategy incentive compatible individually rational mechanism.*

To draw a parallel with the section 2 results, note that the above mechanism is not an efficiency-maximizing Groves mechanism, but it does have certain incentive properties in common with it. First observe that an agent's type report affects his wage only through the allocation decision; within the same allocation of an agent, his wage is constant in his

[22] The finding still holds if the x_{ij}'s are continuous variables.

own report. Indeed, for any $i \in N$ and any $t_{-i} \in T_{-i}$, suppose $x_i^*(t_{-i}, t_i) = x_i^*(t_{-i}, s_i) \equiv x_i^*$ for some $t_i \neq s_i$. Let $t_i > s_i$. Then, by 44,

$$\begin{aligned} & w_i(t_{-i}, t_i) - w_i(t_{-i}, s_i) = \\ & \quad = \sum_{j\in K} c_{ij}(t_i)x_{ij}^* - \int_0^{t_i} \sum_{j\in K} c_{ij}'(\tau)x_{ij}^*(t_{-i}, \tau)\, d\tau - \\ & \quad - \sum_{j\in K} c_{ij}(s_i)x_{ij}^* + \int_0^{s_i} \sum_{j\in K} c_{ij}'(\tau)x_{ij}^*(t_{-i}, \tau)\, d\tau = \\ & \quad = \sum_{j\in K} c_{ij}(t_i)x_{ij}^* - \sum_{j\in K} c_{ij}(s_i)x_{ij}^* - \int_{s_i}^{t_i} \sum_{j\in K} c_{ij}'(\tau)x_{ij}^*(t_{-i}, \tau)\, d\tau = 0\ . \end{aligned} \tag{45}$$

Second, sufficient conditions for monotonicity 35-36 guarantee that, given the allocation rule 43, an agent's true cost report maximizes his own utility as well as the principal's objective function[23]. Comparing these findings with the properties of the DSIC IR mechanisms under complete ignorance, we find that the principal's knowledge of the probability distributions of the agents' cost types – at least in the special case when these types are one-dimensional – is decisive in determining the employment rules which insure that (*i*) individual rationality holds and (*ii*) the expected profit is nonnegative[24] and is maximized. In contrast, under complete ignorance nearly the only way to satisfy individual rationality in DSIC mechanisms was to sacrifice the profit maximization motive and either choose random allocations or pay unreasonably high wages to the agents. One might conclude that no matter how well or poorly informed the agents are about each other's costs, the principal's possession of information is crucial for her profit maximization.

4 A note on Nash equilibrium mechanisms

In the above study, we ignored the case of the extreme informational asymmetry – that is, when the principal is "completely ignorant" (as defined in section 2) but the agents themselves are well-informed about each other's characteristics. Assuming that under this information structure the agents follow Nash equilibrium behavior, we present two

[23]In fact, given any $t_{-i} \in T_{-i}$, i's assignment and, respectively, his wage are step functions of his type report t_i. This follows from the form of the allocation rule 43 and continuity of $J_{ij}(t_i)$ in t_i. Thus, for every $i \in N$ we can identify a collection of threshold types $\{s_{i0}(t_{-i}) = 0, s_{i1}(t_{-i}), ..., s_{iL}(t_{-i}) = \bar{t}_i\}$, $L < \infty$, such that $x_i^*(t_{-i}, t_i)$ and, consequently, $w_i^*(t_{-i}, t_i)$ are constant within each interval $(s_{il-1}(t_{-i}), s_{il}(t_{-i}))$, $l = 1, .., L$. Note that since i's probability of employment is non-deceasing in his type (lemma 3), $x_i^*(t_{-i}, t_i) = 0$ if $t_i \in [0, s_{i1}(t_{-i}))$ and $\sum_{j\in K} x_{ij}^*(t_{-i}, t_i) = 1$ otherwise. Then one can easily show that i's wage at each allocation is a function of his costs at the threshold types only, and does not directly depend on his own type report.

[24]By construction of the optimal mechanism, an allocation $X = 0$ is always an option, which guarantees that the expected profit is nonnegative.

notes on the Nash equilibrium mechanisms. First, we find that if the agents follow Nash equilibrium behavior, the principal can design mechanisms that will always secure her a non-negative profit, but generically cannot guarantee her most preferred outcome. However, we further demonstrate that if the agents are sequentially rational, there exist sequential mechanisms that allow the principal to acquire, almost costlessly, all the hidden information and obtain an outcome which is arbitrarily close to her most preferred alternative. In other words, under certain circumstances the principal can use the agents' self-interest to accumulate the hidden information at a low cost.

4.1 Nash implementation and the first best

Assume that the agents have complete information about each other's types and follow Nash equilibrium behavior. However, the principal has no information about the agents' costs characteristics and can pursue her own interests only by setting the "rules of the game", or the mechanism under which the agents are employed and paid for their jobs. Without focusing on any specific Nash equilibrium mechanism, we use the Nash implementation theory framework to consider what levels of principal's profit are implementable in Nash equilibrium.

As in section 2, let (F, C) characterize an environment, and let $(\mathcal{F}, \mathcal{C})$ be the set of all possible environments, as given in section 2.1. A feasible alternative $a = (X, W)$ is a feasible allocation X and a wage matrix W such that $|\sum_{i \in N} \sum_{j \in K} w_{ij}| \leq M$, where M is a big enough real number[25]. Denote by A the set of all feasible alternatives[26]. The principal's profit corresponding to an alternative a in an environment (F, C), $\pi(a; (F, C))$, is given by 2. With the agents' utility functions given by equation 3, the agents' cost parameters bear sufficient information about their utility functions. Since F is observable for the principal, given any $F \in \mathcal{F}$ we can present a choice rule as a correspondence G_F: $\mathcal{C} \rightarrow A$; denote by $G_F(C)$ the resulting choice set. Then G_F is implementable in Nash equilibrium if there exists a game with the set of Nash equilibria coinciding with the choice set $G_F(C)$.

Consider whether the principal's most preferred choice set is implementable in Nash equilibrium. Since we assume that the agents always have an option not to participate in the game proposed by the principal, a chosen allocation and wage have to be individually rational for every agent. Therefore, in any environment the principal prefers the choice rule that gives her first best, or the complete information outcome given by 9-10. Given

[25]For example, we can choose $M = \sum_i \sum_j \bar{c}_{ij} \bar{x}_{ij}$, where $\bar{c}_i = \sup\{c_i | c_i \in \mathcal{C}_i\}$, and $\bar{X}$ maximizes $F(X) - \sum_i \sum_j \bar{c}_{ij} x_{ij}$. Restrictions on the range of possible wages are imposed for the tractability of analysis; otherwise the agents might unanimously prefer infinitely large wages.

[26]Note that the set of feasible alternatives is not constrained to the set of individually rational alternatives and therefore stays the same for every environment.

$F \in \mathcal{F}$, the principal's first best is the following choice rule:

$$G_F(C) = \begin{cases} X^* & = & X \text{ that maximizes } S(X;(F,C)) \\ w^*_{ij} & = & c_{ij}x^*_{ij} \text{ for all } i,\ j, \end{cases} \tag{46}$$

where, as before, $S(X;(F,C))$ denotes the social surplus of an allocation. We can show that for almost all $(\mathcal{F},\mathcal{C})$ this rule violates the monotonicity property which is necessary for implementability in Nash equilibrium (Maskin, 1979), and thus establish

Proposition 12 *Suppose the set of possible cost profiles $\mathcal{C}$ is such that for every $F \in \mathcal{F}$ there exists $C \in \mathcal{C}$ which satisfies the following conditions: for every surplus-maximizing allocation $X^*(F,C)$ there exists at least one pair (i,j), $i \in N$, $j \in K$ for which $x^*_{ij}(F,C) = 1$ and $c_{ij} > \inf\{c_{ij}|c_{ij} \in \mathcal{C}_{ij}\}$. Then the principal's first best is not implementable in Nash Equilibrium.*

We conclude that under the Nash equilibrium behavior hypothesis, the principal cannot always get her most preferred alternative if she has no information about the environment[27]. The proof of the proposition 12[28] also shows that no matter what Nash equilibrium mechanism the principal uses, under many cost profiles she has to give to the agents substantial shares of social surplus in order to sustain monotonicity. Note, however, that the principal can use her power as mechanism designer to obtain the outcomes which are always individually rational for herself. We introduce the notion of an *acceptable* alternative, which corresponds to the notion of feasibility for the game of surplus redistribution without a principal.

Definition 9 *An alternative $a = (X,W) \in A$ is called acceptable if*

$$F(X) - \sum_i \sum_j w_{ij}x_{ij} \geq 0 \ .$$

Proposition 13 *For any environment, the principal can guarantee an outcome from the set of acceptable alternatives.*

Proof Consider any mechanism that includes the following element. Given observable productivities F, for any (X,W), determined according to some choice rule, let

$$(X^*,W^*) = \begin{bmatrix} (X,W) & \text{if } F(X) - \sum_{i\in N}\sum_{j\in K} w_{ij}x_{ij} \geq 0 \\ (0,0) & \text{otherwise .} \end{bmatrix} \tag{47}$$

[27]The result depends crucially on the assumption that $x_{ij} \in \{0,1\}$, as the proof in section 6 shows.
[28]The proof is given in section 6.

Thus, the principal can "veto" any outcome that gives her a negative payoff by choosing not to employ anybody. □

We summarize the above findings in the following corollary.

Corollary 7 *If the agents follow Nash equilibrium behavior, the principal can guarantee herself a non-negative profit, but cannot guarantee her first best.*

4.2 Sequential mechanisms

Surprisingly, the situation drastically changes in favor of the principal if we assume that the agents follow subgame perfect Nash equilibrium behavior. In this case, the principal can use simple sequential mechanisms to implement the outcomes that in any environment are arbitrarily close to her first best alternative. Indeed, we prove the following proposition.

Proposition 14 *If the agents follow subgame perfect Nash equilibrium behavior, for any arbitrarily small $\epsilon > 0$ there exists an individually rational mechanism $G_\epsilon(F, C) = (X, W)$ such that for any environment (F, C)*

$$\pi(G_\epsilon(F, C)) \geq (1 - \epsilon)\pi^*(F, C) \ ,$$

where $\pi^*(F, C)$ is the principal's complete information profit level.

Proof Consider the following sequential mechanism[29]. For an arbitrary $\epsilon > 0$, choose $\epsilon_1 > 0$, $\epsilon_2 > 0$ so that $\epsilon_1 + \epsilon_2 \leq \epsilon$. Pick randomly two agents $m, l \in N$. Let each stage of the mechanism be observable to the agents. At stage 1, let agent l choose a k-dimensional vector x_m and a scalar s_m, such that x_m constitutes a feasible allocation of agent m. At stage 2, let agent m choose an $(n-1) \times k$ matrix X_{-m} and a $(n-1)$-dimensional vector S_{-m}, where, again, X_{-m} is a feasible allocation of agents other than i. At stage 3, allow any agent to veto m's or l's choices by reporting "no." Define the following "profit" function:

$$p(X, S) \equiv F(X) - \sum_{i \in N} s_i \ . \tag{48}$$

Finally, let the mechanism choose the resulting outcome (X^*, W^*) by the following rule. For every $i \in N$, every $j \in K$, let

$$x^*_{ij} = \begin{cases} 0 & \text{if "no" has been reported by any agent} \\ x_{ij} & \text{otherwise ;} \end{cases} \tag{49}$$

[29] I am grateful to John Duggan for suggesting the idea of this mechanism to me.

$$w_{ij}^* = \begin{cases} s_i & \text{if } x_{ij}^* = 1 \text{ and } i \neq l \text{ , } i \neq m \\ s_l + \epsilon_1 p(X, S) & \text{if } x_{ij}^* = 1 \text{ and } i = l \\ s_m + \epsilon_2 p(X, S) & \text{if } x_{ij}^* = 1 \text{ and } i = m \\ 0 & \text{otherwise .} \end{cases} \tag{50}$$

Using backwards induction reasoning, we now show that any two chosen agents will pick an efficient allocation X and the "base wage" vector S with $s_i = \sum_{j \in K} c_{ij} x_{ij}$ for every $i \in N$. From stage 3, agents l and m cannot be better-off by choosing $s_i < \sum_{j \in K} c_{ij} x_{ij}$ for any i since then their choice will be vetoed and their gain will be identically zero. This guarantees individual rationality of the mechanism. Next, on stage 2 agent m, with his assignment and "base wage" s_m already given, can only maximize his utility by maximizing the principal's profit. This implies that he will choose a surplus-maximizing allocation, constrained to his own allocation, and "base wages" $s_i \leq \sum_{j \in K} c_{ij} x_{ij}$, i.e., $s_i = \sum_{j \in K} c_{ij} x_{ij}$ for every agent including l. Hence at stage 1 agent l knows that no matter what choices he makes, the difference between his "base wage" and cost at his assignment will be identically zero. Therefore, agent l also can increase his utility only through maximizing the principal's profit. It follows that agent l will pick an assignment for m that is consistent with an efficient allocation, and choose $s_m = \sum_{j \in K} c_{mj} x_{mj}$. $\square$

The above analysis indicates that, assuming sequential rationality, a completely uninformed principal can almost costlessly acquire all the information he needs to implement his most preferred outcome. All she needs to do is to hire two informed agents on a profit-sharing basis[30]. One might conclude that the agents do not always gain from having more information: As our results on dominant strategy mechanisms show, if the agents themselves are ignorant of the other agents' cost types, the principal often needs to pay huge information rents to accumulate the dispersed private information. Yet, the agents' complete information case more readily applies to situations where the agents have a past experience of working together than to newly formed teams in flexible organizations.

5 Conclusion

In considering the multiple teams formation problem, we were able to demonstrate several points. First, most generally, the principal's knowledge of the information structure of the agents' characteristics is crucial for her profit-maximization motive – as opposed to efficiency maximization, where no information on the principal's part is required to make an efficient decision. If a principal starts a new project (or a new firm) with no idea how costly this project might be for her, then, even with no nature-induced uncertainty and the information dispersed among the agents, she is likely to run into losses in an attempt to have agents truthfully reveal this information. Yet, if the agents themselves are

[30]Think of foremen who are put in authority over groups of workers or particular operations in a plant.

well-informed about each others' characteristics, the principal can use their self-interest to accumulate this information at a low cost. If the principal is aware of the distribution of the agents' cost characteristics, there exists a well-defined optimal mechanism that maximizes her expected profit. However, when the decision space becomes more complicated, as in the multiproject case, the incentive compatibility constraints are more likely to become binding and thus reduce the principal's profit by often making her treat "good" and "bad" agents equally.

Competition among agents might substitute for the information needed by the principal. We learn that in perfectly competitive environments the principal may collect all the social surplus without having any information on the distribution of the agent's costs. This is also the case when efficiency gains from team production are low. On the other hand, when each agent is indispensable and the efficiency gains from team production are high, the principal is very likely to bear losses. This suggests that a principal might prefer to run a robustly-structured enterprise with homogeneous labor factors and standard tasks rather than a flexible corporation with highly innovative tasks and indispensable agents. Changing to the latter requires acquisition of new information about the agents' characteristics, which might turn out to be very costly for the principal in a hierarchy.

Turning back to our initial question, we find that the appeal for partnerships in the context of flexible organizational forms has theoretical explanations. The agents might want to organize themselves as partners and share efficiency gains from their joint activities when it cannot be profitably done by an outside principal. Yet, incentive compatibility needs to be sustained within a partership, as well as a principal-run firm, if the agents have incomplete information about each other. This important problem has to be addressed before we can argue in favor of partnerships. Still, when the agents are well-informed about each others' characteristics, parterships appear to be a feasible way to achieve higher efficiency by means of flexible organizational forms. Small consulting firms working on a variety of different tasks is the most obvious example.

As the other side of the same conclusion, we might expect large hierarchical structures to have major incentive problems, either on the principals' or on the agents' side, in attempts to reorganize towards more flexible internal structures. A possible solution might be in reducing informational asymmetries or, possibly, changing the ownership structure towards partnerships.

6 Proofs of the statements

Proofs for section 2 *Proof of Lemma 1* Let $\pi^*(F,C)$ denote the profit that the principal would be able to get in an environment (F,C) if she had complete information. Suppose there exists a strongly optimal DSIC IR mechanism $g(F,C)$ such that

$\pi(g(\hat{F},\hat{C})) < \pi^*(\hat{F},\hat{C})$ for some environment $(\hat{F},\hat{C})$. Then consider the following degenerate direct revelation mechanism $\tilde{g}(F,C)$: Denote by $X^*(F)$ an allocation that maximizes

$$F(X) - \sum_{i=1}^{n}\sum_{j=1}^{k} \hat{c}_{ij}x_{ij} \ .$$

Then for any $(F,\tilde{C})$ choose $(\tilde{X}(\tilde{C}),\tilde{W}(\tilde{C}))$ such that

$$\tilde{x}_{ij} = \begin{bmatrix} 1 & \text{if } x^*_{ij}(F) = 1 \text{ and } \tilde{c}_{ij} \leq \hat{c}_{ij} \\ 0 & \text{otherwise} \end{bmatrix} \ ;$$

$$\tilde{w}_{ij} = \begin{bmatrix} \hat{c}_{ij} & \text{if } \tilde{x}_{ij} = 1 \\ 0 & \text{otherwise} \end{bmatrix} \ .$$

Note that, first, $\tilde{g}(F,C)$ is DSIC and IR for any environment and, second, for the environment $(\hat{F},\hat{C})$ it provides the principal the level of profit

$$\pi(\tilde{g}(\hat{F},\hat{C})) = \pi^*(\hat{F},\hat{C}) > \pi(g(\hat{F},\hat{C})) \ .$$

This contradicts our initial supposition that $g(F,C)$ is strongly optimal. □

Proof of Lemma 2 For the class of problems with no production, any efficiency-maximizing DSIC mechanism has to be a Groves mechanism, with the transfers (wages) given in the form (Green and Laffont, 1977):

$$\sum_j w_{ij}x^*_{ij} = -\sum_{l\neq i}\sum_j \tilde{c}_{lj}x^*_{lj}(\tilde{c}_i,\tilde{C}_{-i}) + h(\tilde{C}_{-i}) \ .$$

Introduction of production, however, changes the social efficiency criterion and correspondingly modifies the form of the transfer function. Consider the problem of choosing a socially efficient allocation in the variant with observable production. One can easily show that a direct revelation mechanism is DSIC if and only if it satisfies the following properties (this is a modified "Property A" (Green and Laffont, 1977)):

(i). For all i, w_i is independent of $\tilde{c}_i$ at X^*; i.e., for any F, $\tilde{C}_{-i}$, $\tilde{c}_i$, $\tilde{c}'_i$, if $X^*(\tilde{c}_i,\tilde{C}_{-i},F) = X^*(\tilde{c}'_i,\tilde{C}_{-i},F)$, then $w_i(\tilde{c}_i,\tilde{C}_{-i},F) = w_i(\tilde{c}'_i,\tilde{C}_{-i},F)$.

(ii). $w_i(\tilde{c}_i,\tilde{C}_{-i},F) - w_i(\tilde{c}'_i,\tilde{C}_{-i},F) =$

$$= [F(X^*) - \sum_{l\neq i}\sum_j \tilde{c}_{lj}x^*_{lj}(\tilde{c}_i,\tilde{C}_{-i},F)] - [F(\{\tilde{x}_{ij}\}) - \sum_{l\neq i}\sum_j \tilde{c}_{lj}\tilde{x}_{lj}(\tilde{c}'_i,\tilde{C}_{-i},F)] \ ,$$

where X^* maximizes $F(X) - \sum_i\sum_j \tilde{c}_{ij}x_{ij}(\tilde{c}_i,\tilde{C}_{-i})$, and $\tilde{X}$ maximizes $F(X) - \sum_i\sum_j \tilde{c}_{ij}x_{ij}(\tilde{c}'_i,\tilde{C}_{-i})$.

Next we can show that the only mechanisms that satisfy these properties are the Modified Groves (as defined above). Moreover, all the properties of the standard Groves mechanism hold for the modified version[31]. Therefore, the results regarding the standard Groves mechanisms apply. □

Proof of Proposition 2 By construction, the MPW mechanism is Modified Groves, which implies that it is DSIC and efficient. It is left to show that it is individually rational. Note that if an agent i, $i \in N$, reports the truth, then

$$\sum_{j=1}^{k} w_{ij}^* x_{ij}^* = \max_X [F(X) - \sum_{l \neq i} \sum_j \tilde{c}_{lj} x_{lj} - \sum_j c_{ij} x_{ij}] -$$
$$- \max_{X_{-i}} [F(\{x_{ij}\}_{-i}) - \sum_{l \neq i} \sum_j \tilde{c}_{lj} x_{lj}] + \sum_j c_{ij} x_{ij}^* =$$
$$= S(X^*(\tilde{C}_{-i}, c_i)) - S(\tilde{X}_{-i}(\tilde{C}_{-i})) + \sum_j c_{ij} x_{ij}^* \geq \sum_j c_{ij} x_{ij}^* \tag{51}$$

since

$$S(X^*(\tilde{C}_{-i}, c_i)) \geq S(\tilde{X}_{-i}(\tilde{C}_{-i})) \ .$$

Therefore,

$$\sum_{j=1}^{k} (w_{ij}^* - c_{ij}) x_{ij}^* \geq 0 \ . \tag{52}$$

□

Proof of Proposition 3 As before, let $X^*(F, C)$ denote an efficient allocation of the set N of agent, and $\tilde{X}_{-i}(F, C_{-i})$ – an efficient allocation of the set $\{N \setminus i\}$ agents. Besides, let $N^*(F, C)$ be the set of employed agents: $N^*(F, C) = \{i \in N | \sum_j x_{ij}^* = 1\}$. By corollary 3, it is sufficient to show that for every (F, C)

$$- \sum_{i \in N^*} S(\tilde{X}_{-i}(F, C_{-i}) \leq \sum_{i \in N^*} \tilde{h}_i(C_{-i}) \ , \tag{53}$$

for any $\tilde{h}(C) = (\tilde{h}_1(C_{-1}), .., \tilde{h}_n(C_{-n}))$ such that the corresponding Modified Groves mechanism is individually rational for any (F, C). Suppose this is not the case. Then there exists an environment (F, C) and a vector-function $\tilde{h}(C) = (\tilde{h}_1(C_{-1}), .., \tilde{h}_n(C_{-n}))$, with the corresponding individually rational Modified Groves mechanism $\tilde{g}(F, C)$, such that

$$- \sum_{i \in N^*} S(\tilde{X}_{-i}(F, C_{-i})) > \sum_{i \in N^*} \tilde{h}_i(C_{-i}) \ . \tag{54}$$

[31] The proofs go exactly as they would for the Groves mechanisms and hence do not present anything new of interest; see Green and Laffont (1977) for the original proofs.

This in turn implies that

$$-S(\tilde{X}_{-i}(F,C_{-i})) > \tilde{h}_i(C_{-i}) \text{ for some } i \in N^*. \tag{55}$$

We now show that this leads to the violation of individual rationality in certain environments. Note that since any Modified Groves mechanism is social surplus maximizing, we have

$$S(X^*(F,C)) - S(\tilde{X}_{-i}(F,C_{-i})) \geq 0 \text{ iff } i \in N^*.$$

Two cases are possible:

(i) $S(X^*(F,C)) = S(\tilde{X}_{-i}(F,C_{-i}))$. If condition 55 holds, then individual rationality for i is violated. Hence, this cannot be the case.

(ii) $S(X^*(F,C)) > S(\tilde{X}_{-i}(F,C_{-i}))$. Then suppose $S(X^*) - S(\tilde{X}_{-i}) = a > 0$. In accordance with 55, let $\tilde{h}_i(C_{-i}) = -S(\tilde{X}_{-i}) - \epsilon$ for some $\epsilon > 0$. Then

$$\sum_j w^*_{ij} x^*_{ij} = F(X^*) - \sum_{l \neq i} \sum_j c_{lj} x^*_{lj} + \tilde{h}(C_{-i}) =$$

$$= S(X^*) + \sum_j c_{ij} x^*_{ij} - S(\tilde{X}_{-i}) - \epsilon = \sum_j c_{ij} x^*_{ij} + a - \epsilon$$

and

$$u_i = \sum_j (w^*_{ij} - c_{ij}) x^*_{ij} = a - \epsilon \ .$$

Now consider a different environment $(F, \hat{C})$, such that $\hat{c}_{ij} = c_{ij} + a - \epsilon/2$ for all j, $\hat{c}_{lj} = c_{lj}$ for all $l \neq i$, all j. Then

$$S(X^*(F,\hat{C})) - S(\tilde{X}_{-i}(F,\hat{C}_i)) = \epsilon/2 > 0 \ ,$$

i is still chosen and the whole allocation does not change: $\hat{X} \equiv X^*(F,\hat{C}) = X^*(F,C)$. Further, $\hat{w}_i = w^*_i$, where $\hat{w}_i \equiv w^*_i(F,\hat{C})$, $w^*_i \equiv w^*_i(F,C)$. Then

$$u_i(F,\hat{C}) = \sum_j (\hat{w}_{ij} - \hat{c}_{ij}) \hat{x}_{ij} =$$

$$= \sum_j (w^*_{ij} - c_{ij} - a + \epsilon/2) x^*_{ij} = (a - \epsilon) - a + \epsilon/2 = -\epsilon/2 < O \ , \tag{56}$$

which contradicts individual rationality. □

Proof of Corollary 4 The principal gets a non-negative payoff if $\pi(X^*) = F(X^*) - \sum_i \sum_j w^*_{ij} x^*_{ij} \geq 0$. But

$$F(X^*) - \sum_i \sum_j w^*_{ij} x^*_{ij} =$$

$$= F(X^*) - \sum_i S^* - \sum_i \sum_j c_{ij} x^*_{ij} + \sum_i \tilde{S}_{-i} =$$

$$= (1-n) S^* + \sum_i \tilde{S}_{-i} \ , \tag{57}$$

which is non-negative only if 16 holds. □

Proof of Corollary 5 In this case,

$$\begin{aligned}\pi(X^*) &= F(X^*) - \sum_i \sum_j (w_{ij}^* - c_{ij}) x_{ij}^* = \\ &= \sum_i \tilde{S}_{-i} - (n-1)S^* = n(n-1)\tilde{s}(N_{-i}) - (n-1)ns^*(N) = \\ &= n(n-1)[\tilde{s}(N_{-i}) - s^*(N)] \geq 0 \end{aligned} \tag{58}$$

since $n \geq 1$ and by definition 2. □

Proofs for section 3 *Proof of Proposition 7* Let $U(P, W, s_i | t_i)$ denote i's utility of reporting s_i when his true type is t_i, given the mechanism (P, W). Then one can easily show that

$$U(P, W, s_i | t_i) = U(P, W, s_i) - \sum_{j \in K} (c_{ij}(t_i) - c_{ij}(s_i)) Q_{ij}(P, s_i) \ .$$

If the mechanism (P, W) is BIC, then for any $s_i, t_i \in T_i$

$$U(P, W, t_i) \geq U(P, W, s_i | t_i)$$

and

$$U(P, W, s_i) \geq U(P, W, t_i | s_i)$$

or, equivalently,

$$U(P, W, t_i) \geq U(P, W, s_i) - \sum_{j \in K} (c_{ij}(t_i) - c_{ij}(s_i)) Q_{ij}(P, s_i) \ , \tag{59}$$

$$U(P, W, s_i) \geq U(P, W, t_i) - \sum_{j \in K} (c_{ij}(s_i) - c_{ij}(t_i)) Q_{ij}(P, t_i) \ . \tag{60}$$

It follows that

$$\begin{aligned}&\sum_{j \in K} (c_{ij}(s_i) - c_{ij}(t_i)) Q_{ij}(P, s_i) \leq \\ &\quad \leq U(P, W, t_i) - U(P, W, s_i) \leq \sum_{j \in K} (c_{ij}(s_i) - c_{ij}(t_i)) Q_{ij}(P, t_i) \end{aligned} \tag{61}$$

which yields 29. □

Proof of Proposition 8 For simplicity of notation, let $U(P, W, s_i|t_i) \equiv U(s_i|t_i)$, $U(P, W, t_i) \equiv U(t_i)$. Incentive compatibility means that

$$U(t_i) = \max_{s_i} U(s_i|t_i) \ .$$

From the Envelope theorem, if the mechanism is incentive compatible, then

$$U_i'(t_i) = \frac{\partial U(s_i|t_i)}{\partial t_i} \ ,$$

or

$$U_i'(t_i) = -\sum_{j \in K} c_{ij}'(t_i) Q_{ij}(t_i) \tag{62}$$

for all i, all $t_i \in T_i$. Integrating both sides of the equation and letting $U_i(0) = 0$, we obtain:

$$U_i(t_i) = -\int_0^{t_i} (\sum_{j \in K} c_{ij}'(\tau_i) Q_{ij}(\tau_i)) \, d\tau_i \ . \tag{63}$$

Individual rationality is then guaranteed for all i, all $t_i \in T_i$ since $c_i'(t_i) \leq 0$ by assumption. Let $W_i(t_i)$ denote i's expected wage. Then, by definition,

$$U_i(t_i) = W_i(t_i) - \sum_{j \in K} c_{ij}(t_i) Q_{ij}(t_i) \ ,$$

and therefore the expected wage is

$$W_i(t_i) = \sum_{j \in K} c_{ij}(t_i) Q_{ij}(t_i) - \int_0^{t_i} (\sum_{j \in K} c_{ij}'(\tau_i) Q_{ij}(\tau_i)) \, d\tau_i \ . \tag{64}$$

We now show that 29 and 63 together imply incentive compatibility, i.e.,

$$U(t_i) \geq U(s_i|t_i) \ .$$

for any t_i, s_i. From 63 and 64,

$$U(s_i|t_i) = U(s_i) - \sum_{j \in K} (c_{ij}(t_i) - c_{ij}(s_i)) Q_{ij}(s_i) \ ,$$

and, therefore, it is sufficient to show that for any $s_i < t_i$

$$-\int_{s_i}^{t_i} \sum_{j \in K} c_{ij}'(\tau) Q_{ij}(\tau) d\tau \geq -\sum_{j \in K} (c_{ij}(t_i) - c_{ij}(s_i)) Q_{ij}(s_i) \ ,$$

or

$$-\int_{s_i}^{t_i} \sum_{j \in K} c_{ij}'(\tau) Q_{ij}(\tau) d\tau \geq -\int_{s_i}^{t_i} \sum_{j \in K} c_{ij}'(\tau) Q_{ij}(s_i) d\tau \ .$$

Observe that the above always holds since the necessary condition for monotonicity 29 implies that

$$\sum_{j\in K} c'_{ij}(\tau)(Q_{ij}(\tau) - Q_{ij}(s_i)) \leq 0$$

for all $\tau \geq s_i$. The case $s_i > t_i$ is established by reversing the inequality signs twice.

Incentive compatibility and individual rationality are therefore established. Finally, since

$$\Pi(P) = E_t\{\sum_{l\in L} F_l p_l(t) - \sum_{i\in N}\sum_{j\in K} c_{ij}(t)Q_{ij}(t) - \sum_{i\in N} U_i(t_i)\} ,$$

where E_t denotes expected value over the domain of t, substitution of the expression 63 into the principal's objective function, after standard transformations, yields:

$$\Pi(P) = E_t\{\sum_{l\in L} F_l p_l(t) - \sum_{i\in N}\sum_{j\in K}(c_{ij}(t_i) - c'_{ij}(t_i)\frac{1-H_i(t_i)}{h_i(t_i)})(\sum_{l\in L_{ij}} p_l)\} =$$
$$= E_t\{\sum_{l\in L} F_l p_l(t) - \sum_{i\in N}\sum_{j\in K} J_{ij}(t_i)(\sum_{l\in L_{ij}} p_l)\} . \tag{65}$$

Since both the principal's and the agents' utility functions are linear in allocation X, there cannot be any gain from randomization over X. The optimal choice of $P(t)$ follows. □

Proof of Proposition 9 By proposition 8, the optimal mechanism chooses an allocation $X(t)$ to maximize, subject to the monotonicity constraint,

$$\tilde{S}(X;t) = F(X) - \sum_{i\in N}\sum_{j\in K} J_{ij}(t_i)x_{ij} . \tag{66}$$

We first show that this implies that for any $i \in N$, any s_i, t_i

$$\sum_{j\in K}(J_{ij}(t_i) - J_{ij}(s_i))(x_{ij}(t_i) - x_{ij}(s_i)) \leq 0 . \tag{67}$$

Given the form of the optimal employment rule an agent's probability of being hired is non-decreasing in his type (lemma 3). Thus, we can restrict our attention to the case of an agent being moved from one project to another, within the range of the types where the agent is employed. In the latter case, for an arbitrary $t_{-i} \in T_{-i}$, let X^* be the allocation that maximizes 66 given (t_{-i}, s_i), and $\tilde{X}$ – an allocation that maximizes 66 given (t_{-i}, t_i). Suppose that agent i is optimally employed at project j being of type s_i, but is optimally moved to the project k when his type changes to t_i: $x^*_{ij} = 1$, $x^*_{ik} = 0$

for all $k \neq j$, and $\tilde{x}_{ik} = 1$, $\tilde{x}_{ij} = 0$ for all $j \neq k$. Then

$$\tilde{S}(X^*; t_{-i}, s_i) = F(X^*) - \sum_{l \neq i} \sum_{j \in K} J_{ij}(t_l) x^*_{ij} - J_{ij}(s_i) \geq \tilde{S}(X; t_{-i}, s_i)$$

for any feasible X ;

$$\tilde{S}(\tilde{X}; t_{-i}, t_i) = F(\tilde{X}) - \sum_{l \neq i} \sum_{j \in K} J_{ij}(t_l) \tilde{x}_{ij} - J_{ik}(t_i) \geq \tilde{S}(X; t_{-i}, t_i)$$

for any feasible X .

In particular,

$$F(X^*) - \sum_{l \neq i} \sum_{j \in K} J_{ij}(t_l) x^*_{lj} - J_{ij}(s_i) \geq F(\tilde{X}) - \sum_{l \neq i} \sum_{j \in K} J_{lj}(t_l) \tilde{x}_{lj} - J_{ik}(s_i) \tag{68}$$

and

$$F(\tilde{X}) - \sum_{l \neq i} \sum_{j \in K} J_{lj}(t_l) \tilde{x}_{lj} - J_{ik}(t_i) \geq F(X^*) - \sum_{l \neq i} \sum_{j \in K} J_{lj}(t_l) x^*_{lj} - J_{ij}(t_i) \; . \tag{69}$$

Combining the two above inequalities, we obtain

$$J_{ij}(t_i) - J_{ij}(s_i) \geq J_{ik}(t_i) - J_{ik}(s_i)$$

or

$$\sum_{j \in K} (J_{ij}(t_i) - J_{ij}(s_i))(x_{ij}(t_i, t_{-i}) - x_{ij}(s_i, t_{-i})) \leq 0 \; . \tag{70}$$

Note that interim monotonicity certainly holds if ex-post monotonicity holds, i.e.,

$$\sum_{j \in K} (c_{ij}(t_i) - c_{ij}(s_i))(x_{ij}(t_i, t_{-i}) - x_{ij}(s_i, t_{-i})) \leq 0 \; , \tag{71}$$

which is the case when

$$c_{ij}(t_i) - c_{ij}(s_i) \geq c_{ik}(t_i) - c_{ik}(s_i)$$

whenever

$$J_{ij}(t_i) - J_{ij}(s_i) \geq J_{ik}(t_i) - J_{ik}(s_i) \; .$$

Equivalently, monotonicity holds if

$$c_{ij}(s_i) - c_{ij}(t_i) > c_{ik}(s_i) - c_{ik}(t_i) \tag{72}$$

implies

$$J_{ij}(t_i) - J_{ij}(s_i) < J_{ik}(t_i) - J_{ik}(s_i) \; . \tag{73}$$

But, using the definition of J_{ij}, the latter is always the case if the conditions stated in the proposition hold. $\square$

Proof of Proposition 10 For an arbitrary $i \in N$, take any two $j, k \in K$ and, without loss of generality, suppose (i) is the case. Then for any $s_i, t_i \in T_i$, such that $s_i < t_i$, we have

$$\int_{s_i}^{t_i} c'_{ij}(\tau)\, d\tau < \int_{s_i}^{t_i} c'_{ik}(\tau)\, d\tau \ ,$$

which implies

$$c_{ij}(s_i) - c_{ij}(t_i) > c_{ik}(s_i) - c_{ik}(t_i) \ .$$

Therefore, by proposition 9 it is sufficient to show that for any $s_i, t_i \in T_i$ such that $s_i < t_i$, the following inequality holds:

$$\frac{1 - H_i(t_i)}{h_i(t_i)} c'_{ij}(t_i) - \frac{1 - H_i(s_i)}{h_i(s_i)} c'_{ij}(s_i) \geq \frac{1 - H_i(t_i)}{h_i(t_i)} c'_{ik}(t_i) - \frac{1 - H_i(s_i)}{h_i(s_i)} c'_{ik}(s_i) \ . \tag{74}$$

Note that

$$\frac{1 - H_i(t_i)}{h_i(t_i)} c'_{ij}(t_i) - \frac{1 - H_i(s_i)}{h_i(s_i)} c'_{ij}(s_i) = \int_{s_i}^{t_i} \left(\frac{1 - H_i(\tau)}{h_i(\tau)} c''_{ij}(\tau) + \frac{d}{d\tau}\left(\frac{1 - H_i(\tau)}{h_i(\tau)}\right) c'_{ij}(\tau)\right) d\tau \ .$$

Hence it is sufficient to show that for any $\tau \in [s_i, t_i]$

$$(c''_{ij}(\tau) - c''_{ik}(\tau)) \frac{1 - H_i(\tau)}{h_i(\tau)} + (c'_{ij}(\tau) - c'_{ik}(\tau)) \frac{d}{d\tau}\left(\frac{1 - H_i(\tau)}{h_i(\tau)}\right) \geq 0 \ . \tag{75}$$

Since $(1 - H_i(\tau))/h_i(\tau) \geq 0$ and the Monotone Hazard Rate condition holds, the above inequality follows directly from the assumptions stated in the proposition. □

Proof of Proposition 11 First note that, given that the sufficient conditions for monotonicity hold, the mechanism belongs to the class of optimal BIC mechanisms as defined in proposition 8; hence it is expected profit maximizing. Applying the theorem of Laffont and Maskin (1982) to the team-formation problem, we obtain that an allocation rule $X(t)$ is implementable in dominant strategies if and only if the following conditions are satisfied:

$$\sum_{j \in K} (c_{ij}(t_i) - c_{ij}(s_i))(x_{ij}(t_i) - x_{ij}(s_i)) \leq 0 \ , \tag{76}$$

$$w_i(t_i) = \sum_{j \in K} c_{ij}(t_i) x^*_{ij}(t) - \int_0^{t_i} \sum_{j \in K} c'_{ij}(\tau) x^*_{ij}(t_{-i}, \tau)\, d\tau + e_i(t_{-i})$$
$$\text{for all } i \in N, \text{ all } t_{-i} \in T_{-i} \ , \tag{77}$$

where $e_i(t_{-i})$ is an arbitrary function that does not depend on t_i.

The first condition above is dominant strategy monotonicity; by construction of the sufficient condition for interim monotonicity, it holds whenever the sufficient condition for interim monotonicity holds. In the wage function, using $e_i(t_{-i}) = 0$ guarantees that

$$E_{t_{-i}}(w_i(t_{-i}, t_i)) = \sum_{j \in K} c_{ij}(t_i) Q_{ij}(t_i) - \int_0^{t_i} \sum_{j \in K} c'_{ij}(\tau_i) Q_{ij}(\tau_i)\, d\tau_i$$

for any i, any $t_i \in T_i$, and therefore the principal's expected profit is preserved at the BIC optimal level. Finally, ex-post individual rationality follows from the property that $c'_{ij}(t_i) \leq 0$ for all i, j. □

Proofs for section 4 *Proof of Proposition 12* The principal's role is restricted to the one of mechanism designer, and we only need to prove that the choice function given by 46 is not Nash implementable among the agents. Since monotonicity of a social choice function is a necessary condition for implementability (Maskin, 1979), it is sufficient to show that for any $F \in \mathcal{F}$ $G_F(C)$ given by 46 is non-monotonic. Consider the choice rule given by 46 for an arbitrary $F \in \mathcal{F}$. If $G_F(C)$ is monotonic, then for any C, $\tilde{C}$, if $a \in G_F(C)$ and for any $b \in A$, for any i, $u_i(a; C) \geq u_i(b; C)$ implies $u_i(a; \tilde{C}) \geq u_i(b; \tilde{C})$, then $a \in G_F(\tilde{C})$. Take an arbitrary cost profile $C \in \mathcal{C}$ such that for every surplus-maximizing allocation $X^*(F, C)$ there exists at least one pair (i, j), $i \in N$, $j \in K$ for which $x^*_{ij}(F, C) = 1$ and $c_{ij} > \inf\{c_{ij} | c_{ij} \in \mathcal{C}_{ij}\}$ (under the conditions specified in the statement above, such C always exists). Let an alternative $a = (X^*, W^*)$, as defined in 46 be in the choice set, $(X^*, W^*) \in G_F(C)$. Let $N_1 \subseteq N$ denote the set of agents i employed in X^* for which there exist a project $j(i)$ such that $x^*_{ij}(F, C) = 1$ and $c_{ij} > \inf\{c_{ij} | c_{ij} \in \mathcal{C}_{ij}\}$. Note that N_1 is always non-empty by the assumption of the proposition. Next, consider another environment $\tilde{C}$ such that $\tilde{c}_{ij} = c_{ij} - \epsilon$ if $i \in N_1$ and $x^*_{ij} = 1$, for some $\epsilon > 0$ small enough to ensure that $\tilde{c}_{ij} \in \mathcal{C}_{ij}$, and $\tilde{c}_{ij} = c_{ij}$ otherwise. Then for any $b \in A$, $u_i(b; \tilde{C}) = u_i(b; C) + \epsilon \sum_j x_{ij}(b)$ if $i \in N_1$ and $x_{ij}(b) = x^*_{ij}$, and $u_i(b; \tilde{C}) = u_i(b; C)$ otherwise; note also that $x_{ij} \in \{0, 1\}$ for all i, j. Therefore, for any $b \in A$, any $i \in N$, $u_i(a; C) \geq u_i(b; C)$ implies $u_i(a; \tilde{C}) \geq u_i(b; \tilde{C})$; then monotonicity requires that $a \in G_F(\tilde{C})$[32]. However, 46 requires that $u_i(a; \tilde{C}) = 0$ if $a \in G_F(\tilde{C})$ which obviously does not hold. Therefore, $a \notin G_F(\tilde{C})$ and the necessary monotonicity condition is violated under 46. □

[32]Notice that the assumption that $x_{ij} \in \{0, 1\}$ is crucial for the analysis: Suppose that $0 < x_{ij}(a) < 1$ for some (i, j). Then consider an alternative b such that $x_{ij}(b) = 1$ for this (i, j), and $w_i(b) = c_{ij} - \epsilon_1$, where $0 < \epsilon_1 < \epsilon$. Then $u_i(a; C) = 0 > u_i(b; C)$. However, $u_i(a; \tilde{C}) = \epsilon * x_{ij}(a)$, whereas $u_i(b; \tilde{C}) = \epsilon_1$. Thus, if $\epsilon_1 > \epsilon * x_{ij}(a)$, we obtain that $u_i(a; \tilde{C}) < u_i(b; \tilde{C})$.

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